Methods for Degenerate Nonlinear Programming

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5th MOPTA Conference Modeling and Optimization: Theory and Applications

July 25-27, 2005 - University of Windsor, Windsor ON Canada

Outline

- Background
- Problem statement
- Motivation
- Optimality conditions and Lagrange–Newton method
- P-factor-Lagrange method
- Nonlinear complementarity problems
- Conclusion

Background

Consider the nonlinear programming (NLP) problem:

$$\begin{array}{ll} \underset{x \in \mathbb{R}^n}{\text{minimize}} & f(x) \\ \text{subject to} & g_1(x) \le 0, \ldots, g_m(x) \le 0, \end{array}$$

where f and g_j are smooth functions from $\mathbb{R}^n$ to $\mathbb{R}$.

We focus on finding a *local solution* x^* of the NLP.

The *Lagrangian* for the NLP problem is defined as

$$\mathcal{L}(x, \lambda) = f(x) + \sum_{j=1}^{m} \lambda_j g_j(x),$$

where $\lambda = (\lambda_1, \ldots, \lambda_m)$ is the vector of Lagrange multipliers.

Constraint qualification

Introduce the set of *active constraints:*

$$A(x^*) = \{j = 1, \ldots, m \,|\, g_j(x^*) = 0\}.$$

Constraint qualifications:

- *Linear Independence Constraint Qualification (LICQ)*: The gradients $\nabla g_j(x^*),\ j \in A(x^*)$, are linearly independent.

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- *Mangasarian-Fromovitz Constraint Qualification (MFCQ)*: There is a direction d such that $\nabla g_j(x^*)^T d < 0$ for all $j \in A(x^*)$.

First-Order Necessary Conditions

If x^* is a local minimizer of the NLP problem and a CQ is satisfied at x^*, then there is $\lambda^* \in \mathbb{R}^m$ such that

$$\nabla_x \mathcal{L}(x^*, \lambda^*) = \nabla f(x^*) + \sum_{j=1}^{m} \lambda_j^* \nabla g_j(x^*) = 0$$

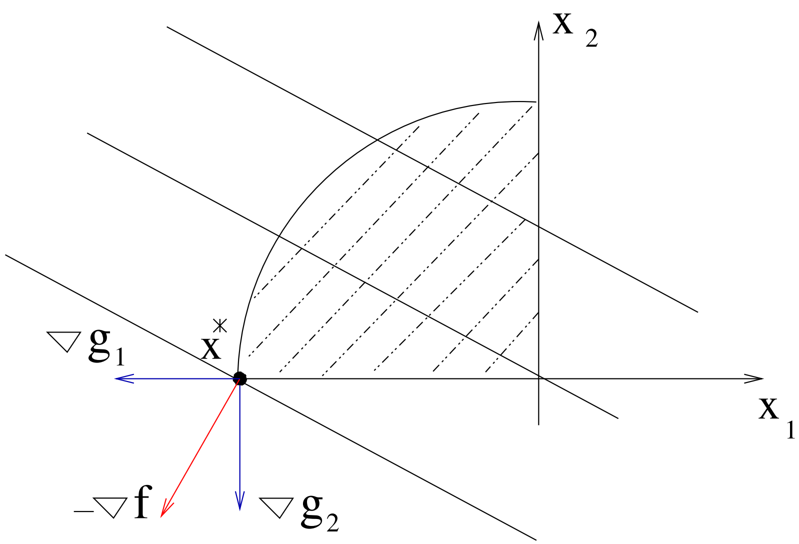
$$g(x^*) \leq 0, \quad \lambda^* \geq 0, \quad g_j(x^*)\lambda_j^* = 0, \quad j = 1, \dots, m$$

“Objective anti-gradient is a conic combination of the active constraint gradients"

Question 1: How to use the optimality conditions to solve the NLP?

Conditions for Optimality: *Example*

$$\begin{array}{ll} \underset{x\in\mathbb{R}^2}{\text{minimize}} & f(x_1,x_2) = x_1 + 2x_2 \\ \text{subject to} & g_1(x_1,x_2) = x_1^2 + x_2^2 - 2 \le 0 \\ & g_2(x_1,x_2) = -x_2 \le 0, \quad g_3(x_1,x_2) = x_1 \le 0 \end{array}$$



The minimizer $x^* = (-\sqrt{2}, 0)$ satisfies

$$-\nabla f(x^*) = \lambda_1^* \nabla g_1(x^*) + \lambda_2^* \nabla g_2(x^*), \quad \lambda_1^* \ge 0, \quad \lambda_2^* \ge 0.$$

Standard Assumptions

Local convergence analysis of algorithms *usually assumes*

- Constraint qualification (LICQ or MFCQ);
- Strict complementarity condition (SCC): exactly one of λ_j^* and $g_j(x^*)$ is zero for each index $j = 1, \ldots, m$.
- The second-order sufficient condition (SOSC): there exists $\nu > 0$ such that

$$\omega^T \mathcal{L}_{xx}(x^*, \lambda^*)\omega > \nu \|\omega\|^2$$

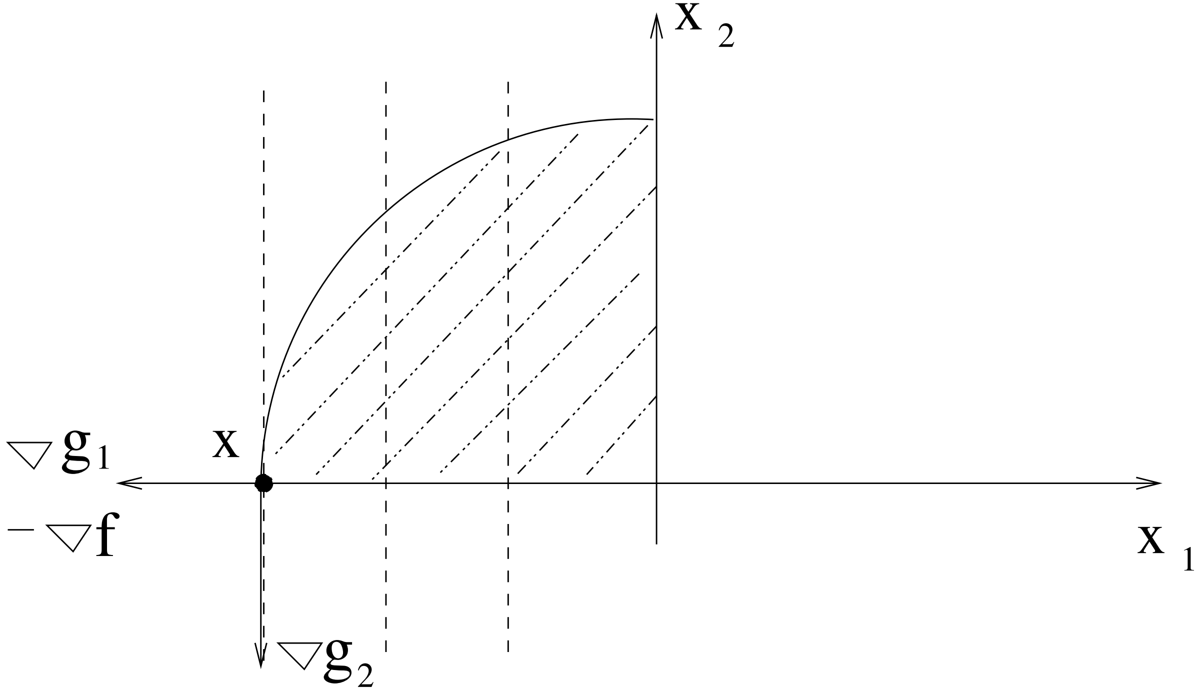
for all ω such that $\nabla g_j(x^*)^T \omega = 0, \quad$ for all $\quad j \in A(x^*)$.

Question 2: Are these assumptions always satisfied?

Example: No SCC

SCC: $\lambda_j^* > 0$ for each $g_j(x^*) = 0$.

Example. The SCC *does not hold:* $\lambda_2^* = 0, \; g_2(x^*) = 0.$



$$-\nabla f(x^*) = \lambda_1^* \nabla g_1(x^*) + 0 \nabla g_2(x^*)$$

Example: No SOSC

There are problems where the SOSC *does not hold*:

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & x^4 \\ \text{subject to} \quad & x^3 \leq 0. \end{aligned}$$

In this problem $x^* = 0$ and

$$\mathcal{L}(x,\lambda) = x^4 + \lambda x^3, \quad \mathcal{L}_x(x,\lambda) = 12x^2 + 6\lambda x, \quad \mathcal{L}_{xx}(x^*,\lambda^*) = 0.$$

Thus, the SOSC is not satisfied at the point $(0, \lambda^*)$:

$$\omega^T \mathcal{L}_{xx}(x^*,\lambda^*)\omega = 0 \leq \nu\|\omega\|^2, \quad \nu > 0.$$

Problem statement

Find a local solution x^* of NLP in which

- the strict complementarity condition,
- a constraint qualification,
- a second-order sufficient condition for optimality

are *not* necessarily satisfied at the solution.

Motivation and Goals

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Goals:

- Propose new optimality conditions for some classes of NLP problems
- Design algorithms that perform as well as possible in a variety of difficult circumstances

Conversion to the Equality Case

Convert the NLP problem into the equality constrained problem:

$$\begin{array}{ll} \underset{(x,s)}{\text{minimize}} & f(x) \\ \text{subject to} & g_1(x) + s_1^2 = 0, \ldots, g_m(x) + s_m^2 = 0, \end{array}$$

where $s_1, \ldots, s_m$ are additional variables.

Define the *Lagrangian function* as

$$L(x, s, \lambda) = f(x) + \sum_{j=1}^{m} \lambda_j (g_j(x) + s_j^2).$$

Lagrange system

If x^* is a minimizer, then (x^*, s^*, λ^*) is the solution to the *Lagrange system*:

$$F(x, s, \lambda) = \begin{bmatrix} L_x(x, s, \lambda) \\ 2\lambda s \\ g(x) + s^2 \end{bmatrix} = 0.$$

Lagrange-Newton method is the Newton method applied to the Lagrange system:

$$z^{k+1} = z^k - J(z^k)^{-1} F(z^k), \quad z = (x, s, \lambda).$$

Question 3: Where is the problem?

Lagrange–Newton Method

Write the Jacobian $J = F'(x, s, \lambda)$ explicitly:

$$
J = \left[\begin{array}{c|ccc|ccc}
L_{xx} & & 0 & & & \nabla g(x) & \\
\hline
 & 2\lambda_1 & \dots & 0 & 2s_1 & \dots & 0 \\
0 & \dots & \dots & \dots & \dots & \dots & \dots \\
 & 0 & \dots & 2\lambda_m & 0 & \dots & 2s_m \\
\hline
 & 2s_1 & \dots & 0 & & & \\
\nabla g(x)^T & \dots & \dots & \dots & & 0 & \\
 & 0 & \dots & 2s_m & & &
\end{array}\right]
$$

Problem: The *matrix J is singular* at (x^*, s^*, λ^*) if one of the standard assumptions (LICQ, SCC, SOSC) is not satisfied.

Other work in the area

- Under assumptions of SOSC and of existence of a KKT point, Izmailov and Solodov (2005) and Wright (2005) proposed an approach that is based on the correct identification of the active constraint set $A(x^*)$.

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- the set $A(x^*)$ is hard to identify for the degenerate problems;
- Jacobian of the Newton system is singular in the limit;
- What if the SOSC does not hold?

Reformulation of the problem

Consider the system of equations

$$F(x) = \begin{pmatrix} F_1(x) \\ \dots \\ F_n(x) \end{pmatrix} = 0, \quad x \in \mathbb{R}^n.$$

Goal: Construct a local method for solving $F(x) = 0$ with a superlinear rate of convergence under assumption that the Jacobian matrix $F'(x^*)$ may be singular at a solution x^*, i.e.,

$$\operatorname{rank} F'(x^*) = \operatorname{rank} \begin{pmatrix} \nabla F_1^T(x^*) \\ \dots \\ \nabla F_n^T(x^*) \end{pmatrix} < n.$$

Nonlinear Equations

Notation:

For a given linear operator $A : \mathbb{R}^n \to \mathbb{R}^n$,

$$\operatorname{Im} A = \{y \in \mathbb{R}^n \,|\, y = Ax \text{ for some } x \in \mathbb{R}^n\}$$

denotes the image space.

$(\operatorname{Im} F'(x^*))^\perp$ is an orthogonal complementary subspace to the image of the first derivative of F;

$P^\perp$ is a matrix of the orthoprojector onto $(\operatorname{Im} F'(x^*))^\perp$.

Assumption:

$$\operatorname{Im}(F'(x^*) + P^\perp F''(x^*)h) = \mathbb{R}^n, \quad h \in \mathbb{R}^n.$$

2-factor-method

2-factor-method is the following iterative method:

$$x^{k+1} = x^k - \left[F'(x^k) + P^{\perp} F''(x^k)[h]\right]^{-1} \left[F(x^k) + P^{\perp} F'(x^k)[h]\right],$$

Remark: 2-factor-method is an application of the Newton method to the system $\Phi(x) = F(x) + P^{\perp} F'(x)[h] = 0$.

Under our assumption, there is a neighborhood $U(x^*)$ of x^* in $\mathbb{R}^n$ such that for any $x^0 \in U(x^*)$, the 2-factor-method converges to x^* and $\|x^{k+1} - x^*\| \leq C\|x^k - x^*\|^2, \quad C > 0.$

Remark: For details about 2-factor-method see Izmailov and Tret'yakov (1999) or Tret'yakov and Marsden (2003); about singular nonlinear equations – Brezhneva and Izmailov (2002).

2-factor-Lagrange Method

Recall that we consider the following problem:

Problem: Solve the Lagrange system

$$F(x, s, \lambda) = 0$$

for which the Jacobian matrix $F'(x^*, s^*, \lambda^*)$ might be *singular*.

2-factor-Lagrange method: Apply 2-factor-method to solve the Lagrange system. First, we will construct

$$\Phi(x, s, \lambda) = F(x, s, \lambda) + P^{\perp}(F'(x, s, \lambda)h)$$

such that if (x^*, s^*, λ^*) is a solution to $\Phi(x, s, \lambda) = 0$, then x^* is a local minimizer to the NLP problem.

Assumption

Assumption A1. If x^* is a solution of the NLP then for some λ^* and $z^* = (x^*, s^*, \lambda^*)$ the following condition holds:

$$\operatorname{Im}(F'(z^*) + P^{\perp} F''(z^*)h) = \mathbb{R}^{n+2m}, \quad h \in \mathbb{R}^{n+2m}.$$

Lemma. Let x^* be a solution to the NLP problem and the necessary optimality conditions are satisfied with some Lagrange multiplier λ^*.
If the standard second-order sufficient condition (SOSC) holds at x^*, then Assumption A1 is satisfied.

Constructing Φ

Assume that estimates of both x and λ are available.

First technique: Using singular-value decomposition of the matrix $F'(x, s, \lambda)$.

Second technique: Using an estimate of the set of the linearly independent gradients $\nabla F_1(z^*), \ldots, \nabla F_r(z^*)$, $r < n$.

The following defines a computable estimate of the distance to solution set $S = \{z \in \mathbb{R}^{n+2m} \mid F(z) = 0\}$:

$$\mu(z) \approx dist(z, S), \quad \mu(z) = \|F(z)\|^{1/2}.$$

Constructing Φ

Let $\mu = C\mu^{1/2}(z)$, $C > 0$. Then

1) The vectors $\nabla F_{i_1}(z^*), \dots, \nabla F_{i_s}(z^*)$ are linearly independent iff

$$\mathrm{dist}(\nabla F_{i_k}(z), L^s_{i_k}(z)) \geq \mu \quad \forall k = 1, \dots, s, \quad \forall z \in U(z^*),$$

where $L^s_{i_k}(z)$ is the linear span, $s \geq 2$,

$$L^s_{i_k}(z) = \mathrm{span}\{\nabla F_{i_1}(z), \dots, \nabla F_{i_{k-1}}(z), \nabla F_{i_{k+1}}(z), \dots, \nabla F_{i_s}(z)\}.$$

2) $\nabla F_i(z^*) = 0$ iff

$$\|\nabla F_i(z)\| < \mu, \quad \forall z \in U(z^*).$$

Constructing Φ

Assume that the first r vectors $\nabla F_1(z^*), \ldots, \nabla F_r(z^*)$ are linearly independent, and that others are zeros, i.e, $\nabla F_{r+1}(z^*) = 0$, ..., $\nabla F_{n+2m}(z^*) = 0$. If they were not zeros, we would transform the original system into another one that has the same solution z^* and that satisfies these assumptions.

Then $P^\perp$, which is the orthoprojector onto $(\operatorname{Im} F'(z^*))^\perp$, is a diagonal matrix $P^\perp = \operatorname{diag}(p_j)_{j=1}^{n+2m}$ that is given by

$$p_j = \begin{cases} 0, & j = 1, \ldots, r \\ 1, & j = r+1, \ldots, n+2m \end{cases}.$$

Constructing Φ

We get the mapping $\Phi(z)$:

$$\Phi(z) = F(z) + P^{\perp} F'(z)[h],$$

where the choice of the vector $h \in \mathbb{R}^{n+2m}$ is flexible and should be done in such a way that

$$\mathrm{rank}(\Phi'(z^*)) = n + 2m.$$

Having the mapping $\Phi(z)$ and a point z^0, we can define the 2-factor-Lagrange method:

$$z^{k+1} = z^k - \left[\Phi'(z^k)\right]^{-1} \left[\Phi(z^k)\right], \quad k = 0, 1, 2, \ldots$$

Theorem

Let x^* be a solution of NLP. Let $f, g \in C^3(\mathbb{R}^n)$ and let $z^* = (x^*, s^*, \lambda^*)$ satisfy the necessary optimality conditions:

$$F(x, s, \lambda) = \nabla L(x, s, \lambda) = 0.$$

Assume that Assumption A1 holds:

$$\mathrm{Im}(F'(z^*) + P^{\perp} F''(z^*)h) = \mathbb{R}^{n+2m}, \quad h \in \mathbb{R}^{n+2m}.$$

Then there is a neighborhood $V(x^*)$ of z^* in $\mathbb{R}^{n+2m}$ such that for any $z^0 \in V(z^*)$, the 2-factor-Lagrange-method converges to z^* and

$$\|z^{k+1} - z^*\| \leq C\|z^k - z^*\|^2, \quad C > 0.$$

Example

Consider the problem of minimizing the functional

$$J_0[y] = \int_{-3/2}^{3/2} y^2 dx$$

subject to the constraints

$$\int_{-3/2}^{3/2} ((y')^2 - y^2)dx \leq 0, \quad y(-3/2) = y(3/2) = 0.$$

Example

To discretize the problem, introduce the notation:

$$x_1 = y(-1/2), \quad x_2 = y(1/2),$$

and use the trapezoidal rule to approximate the integrals.

We get the following problem:

$$\begin{array}{ll} \underset{x \in \mathbb{R}^2}{\text{minimize}} & f(x_1, x_2) = x_1^2 + x_2^2 \\ \text{subject to} & g(x_1, x_2) = 3x_1^2 + x_2^2 - 4x_1 x_2 \leq 0 \end{array}$$

The solution to this problem is $x^* = (0,0)^T$.

Example

Reduce the problem to the equality constrained problem

$$\begin{array}{ll} \text{minimize} & x_1^2 + x_2^2 \\ \text{subject to} & 3x_1^2 + x_2^2 - 4x_1x_2 + s^2 = 0 \end{array}$$

The Lagrangian is

$$L(x_1, x_2, s, \lambda) = x_1^2 + x_2^2 + \lambda(3x_1^2 + x_2^2 - 4x_1x_2 + s^2).$$

The first-order conditions can be stated as

$$F(x, s, \lambda) = \begin{bmatrix} 2x_1 + 6\lambda x_1 - 4\lambda x_2 \\ 2x_2 + 2\lambda x_2 - 4\lambda x_1 \\ 2\lambda s \\ 3x_1^2 + x_2^2 - 4x_1x_2 + s^2 \end{bmatrix} = 0.$$

Example

The Jacobian of F is defined as

$$F'(x, s, \lambda) = \begin{bmatrix} 2+6\lambda & -4\lambda & 0 & 6x_1 - 4x_2 \\ -4\lambda & 2+2\lambda & 0 & 2x_2 - 4x_1 \\ 0 & 0 & 2\lambda & 2s \\ 6x_1 - 4x_2 & 2x_2 - 4x_1 & 2s & 0 \end{bmatrix}.$$

This matrix is singular at $x^* = (0,\ 0)^T$ with $\lambda^* = 0$ and $s^* = 0$. We apply the described method with $h = (0, 0, 1, 1)^T$ to get

$$\Phi(z) = F(z) + P^{\perp} F'(z)[h]$$

Example

The idea behind the algorithm is that instead of solving Lagrange system, which has the singular Jacobian, we are solving the following system:

$$\Phi(x, s, \lambda) = \begin{bmatrix} 2x_1 + 6\lambda x_1 - 4\lambda x_2 \\ 2x_2 + 2\lambda x_2 - 4\lambda x_1 \\ 2\lambda s + 2\lambda + 2s \\ 3x_1^2 + x_2^2 - 4x_1 x_2 + s^2 + 2s \end{bmatrix} = 0.$$

with the nonsingular Jacobian.
The system $\Phi = 0$ has the same solution $(0, 0, 0, 0)^T$ as the original system.

Example

At each iteration, the step $(\Delta x, \Delta s, \Delta\lambda)$ is a solution of the following system

$$\begin{bmatrix} 2+6\lambda & -4\lambda & 0 & 6x_1-4x_2 \\ -4\lambda & 2+2\lambda & 0 & 2x_2-4x_1 \\ 0 & 0 & 2\lambda+2 & 2s+2 \\ 6x_1-4x_2 & 2x_2-4x_1 & 2s+2 & 0 \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \Delta s \\ \Delta\lambda \end{bmatrix}$$

$$= - \begin{bmatrix} 2x_1+6\lambda x_1-4\lambda x_2 \\ 2x_2+2\lambda x_2-4\lambda x_1 \\ 2\lambda s+2\lambda+2s \\ 3x_1^2+x_2^2-4x_1x_2+s^2+2s \end{bmatrix}.$$

NCP

Given a (smooth) mapping $g(x) : \mathbb{R}^n \rightarrow \mathbb{R}^n$. Find $x \in \mathbb{R}^n$ such that

$$g(x) \leq 0, \quad x \leq 0, \quad x^T g(x) = 0.$$

A. Economic Applications:

- General equilibrium problems
- Invariant capital stock problems
- Spatial price equilibrium problems

B. Physical (Engineering) Applications:

- Contact mechanics problems
- Obstacle (and free boundary) problems
- Traffic equilibrium problems

Method for Solving NCP

Convert the NCP problem into

$$F(x,s,y) = \begin{pmatrix} g(x) + s^2 \\ x + y^2 \\ sy \end{pmatrix} = 0.$$

The corresponding Jacobian is given by

$$F'(x,s,y) = \begin{pmatrix} \nabla g(x) & 2S & 0 \\ I & 0 & 2Y \\ 0 & Y & S \end{pmatrix},$$

where $S = \mathrm{diag}(s_j)_{j=1}^m$, $Y = \mathrm{diag}(y_j)_{j=1}^m$, and I is the identity matrix.

Method for Solving NCP

Introduce the set $J_0 = \{j = 1, \ldots, m \,|\, y_j^* = 0,\ s_j^* = 0\}$.
Define the vector $h = (h_1, \ldots, h_n)$ as

$$h_i = \begin{cases} 1, & i \in J_0 \\ 0, & i \notin J_0 \end{cases}$$

and a vector $\bar{h} \in \mathbb{R}^{3n}$ as $\bar{h} = (0, 0, h)$.
In NCP, the orthoprojector $P^\perp$ onto $(\operatorname{Im} F'(x, s, y))^\perp$ is a constant diagonal matrix $P^\perp = \operatorname{diag}(p_j)_{j=1}^{3n}$:

$$p_i = \begin{cases} 1, & i = n + j, \quad \text{where} \quad j \in J_0 \\ 1, & i = 2n + j, \quad \text{where} \quad j \in J_0 \\ 0, & \text{otherwise.} \end{cases}$$

Method for Solving NCP

Assume for simplicity that $J_0 = \{1, \ldots, n\}$. Then we get

$$F(x,s,y) + P^{\perp} F'(x,s,y)\bar{h} = \begin{pmatrix} g + s^2 \\ x + y^2 + 2y \\ sy + s \end{pmatrix}$$

and

$$\begin{pmatrix} x^{k+1} \\ s^{k+1} \\ y^{k+1} \end{pmatrix} = \begin{pmatrix} x \\ s \\ y \end{pmatrix} - \begin{bmatrix} (\nabla g(x))^T & 2S & 0 \\ I & 0 & 2(Y+I) \\ 0 & Y+I & S \end{bmatrix}^{-1}$$

$$\begin{pmatrix} g(x) + s^2 \\ x + y^2 + 2y \\ sy + s \end{pmatrix}.$$

Conclusion

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- We proposed a new method for solving the Lagrange optimality system whose Jacobian might be singular at the solution.
- We illustrated the proposed approach by constructing a method for nonlinear complementarity problems (NCP).

References

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- More general approach to solving singular nonlinear equations is described in
  [5] O. Brezhneva and A. Izmailov (2002), *Construction of defining systems for finding singular solutions to nonlinear equations*, Comput. Math. and Math. Phys., 42, pp. 8–19.

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  [5] O. Brezhneva and A. Izmailov (2002), *Construction of defining systems for finding singular solutions to nonlinear equations*, Comput. Math. and Math. Phys., 42, pp. 8–19.
- This talk presents the paper:
  [6] O. Brezhneva and A. Tret'yakov, P*-factor-Lagrange methods for degenerate nonlinear programming*, in preparation.

End...

Thank you!