Price Guarantees in Dynamic Pricing and Revenue Management

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Background — Price Guarantees

- very common in retail sector
  - internal or external
  - forced by ‘free returns’ policies
  - typically free of charge
- potential benefits
  - customer: reduced risk of opportunity loss
  - company: encourage immediate purchase, improve customer satisfaction
- surprisingly little prior analytical work

Motivation

- emergence of dynamic pricing
- nondecreasing fares in refundable fare classes
- variety of possible forms
  - internal price-matching guarantee?
  - partial price-matching *strike price*?
  - fee for guarantee?

Potential Scope of Price Guarantee Tools

- price matching in retail industry
- real estate
- car rentals
- hotel and motel accommodations
- concert and sport events
- airline tickets
- space launch services!

Related Work

- Dynamic pricing:
  - Gallego and van Ryzin (1994)
  - Feng and Xiao (2000a,2000b)
  - Zhao and Zheng (2000)
  - Chatwin (2000)
- Marketing and economics:
  - Hess and Gerstner (2001)
  - Moorthy and Winter (2002)
  - Srivastava and Lurie (2001)

Elements of Formulation

- finite time horizon: $[0, T]$
- policy variables
  - dynamic prices: $p(t)$
  - guarantee strike price: $k(t)$
  - fee for guarantee: $f(t)$
- demand processes
  - Poisson inquiry process: $N(t)$, rate λ
  - probability of item purchase: $u[p(t), k(t), f(t), t]$
  - item purchase process: $N_p(t)$, rate $\lambda u[\cdot]$
  - probability of guarantee purchase : $v[p(t), k(t), f(t), t]$
  - guarantee purchase process: $N_f(t)$, rate $\lambda u[\cdot] v[\cdot]$

Demand Response Model — Probabilities u and v

- most results based on general assumptions about u and v
- strike price ratio $\kappa = k/p$
- fee ratio $\phi = f/k$
- some qualitative assumptions:
  - u and v increasing in κ, decreasing in ϕ
  - u decreasing in p and $u \to 0$ as $p \to \infty$
  - $v \to 0$ as $\phi \to 1$ or $\kappa \to 0$
  - $v \to 1$ as $\phi \to 0$
  - u time dependent and less sensitive to ϕ and κ as $t \to T$
  - v decreasing in time and $v \to 0$ as $t \to T$

The objective is to maximize

Revenue due to item sales

$$E\left[\int_0^T p(s)dN_p(s)\right]$$

plus revenue due to sales of price guarantees

$$E\left[\int_0^T f(s)dN_f(s)\right]$$

minus losses due to compensation payments

$$-E\left[\int_0^T \max_{\tau\in[s,T]}(k(s)-p(\tau))^+dN_f(s)\right]$$

Policies

- policy is a triple $\{p(t), k(t), f(t)\}$ for $t \in [0, T]$
- dependent only on the history of the demand and fare processes up to time t
- non-randomized

Challenges

- carrying price guarantees influences terminal rewards
- must track entire history?
  - state space explosion
- model is non-Markovian
  - impractical to use conventional state-space augmentation

Approach

- formulated complete continuous time problem
- analyzed discrete-time analogue
- developed nonlinear programming approach to solution
- studied structural properties of the model
- developed lower bound heuristic
- numerical experiments with discrete model
  - exact for small problems – NLP
  - lower bound heuristic for larger problems via dynamic programming

Probability Space

- point processes

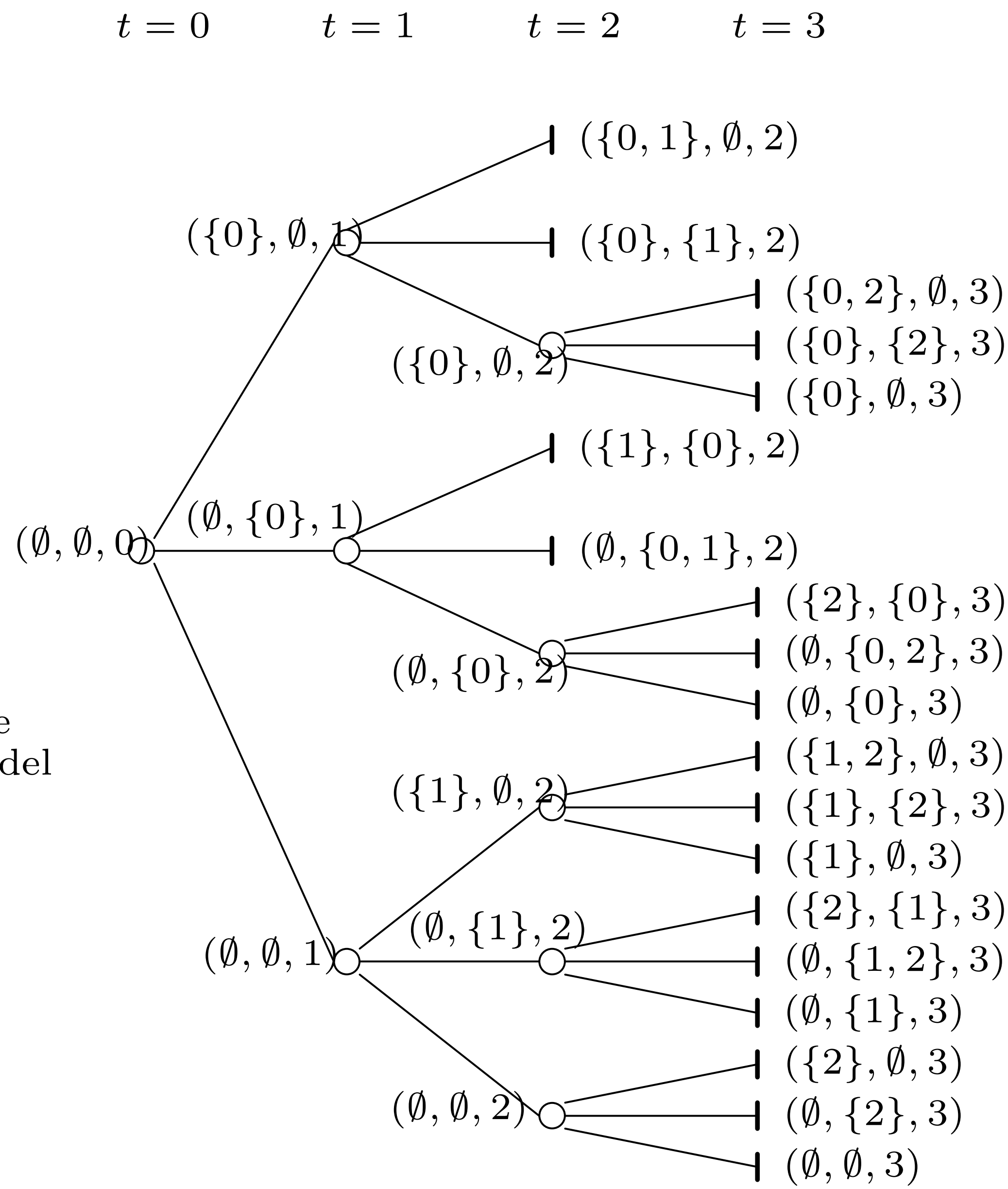
$$N_1(t) = N_p(t) - N_f(t),$$
$$N_2(t) = N_f(t).$$

- jump times up to time t

$$\mathcal{N}_1(t) = \{t_{11}, t_{12}, \ldots, t_{1N_1(t)}\}, \text{ and}$$
$$\mathcal{N}_2(t) = \{t_{21}, t_{22}, \ldots, t_{2N_2(t)}\}$$

- history up to time t : $(\mathcal{N}_1, \mathcal{N}_2, t)$
- price, strike price, fee processes

$$p(\mathcal{N}_1, \mathcal{N}_2, t),\ k(\mathcal{N}_1, \mathcal{N}_2, t),\ f(\mathcal{N}_1, \mathcal{N}_2, t)$$



Probability Tree
for Discrete Model

$Y = 2, T = 3$

Discrete Model Formulation

- unnecessary to include the history of the fare process in the state representation, unless one uses dynamic programming

- optimize all variables simultaneously $\Longrightarrow$ policies depend only on t, and times of sales with and without price guarantees

Main Results

- existence of optimal policy
- necessary optimality conditions via NLP formulation
- intuitive monotonicity results for value function
- in case of free price guarantee the demand is more sensitive to changes in price than in strike price
- sufficient conditions on demand levels/promotional effects for fixed policy with price guarantees to dominate dynamic policy without price guarantees
- useful lower bound heuristic

Myopic Lower-bounding Heuristic

- exact model is solvable only for limited size problems
- for each history of the sales process, maximize the expected total of the current and future revenues, including price guarantee payments for the current and future guarantee sales but disregarding payments for all *previously sold* guarantees
- implementable via dynamic programming

Demand Functions for Numerical Examples

- strike price ratio $\kappa = k/p$
- fee ratio $\phi = f/k$

$$u(p,\kappa,\phi,t) = \exp\left\{-p - \alpha\left[1 - \kappa^{\beta'}(1-\phi)^{\gamma'}v_0(t)\right]\right\}$$

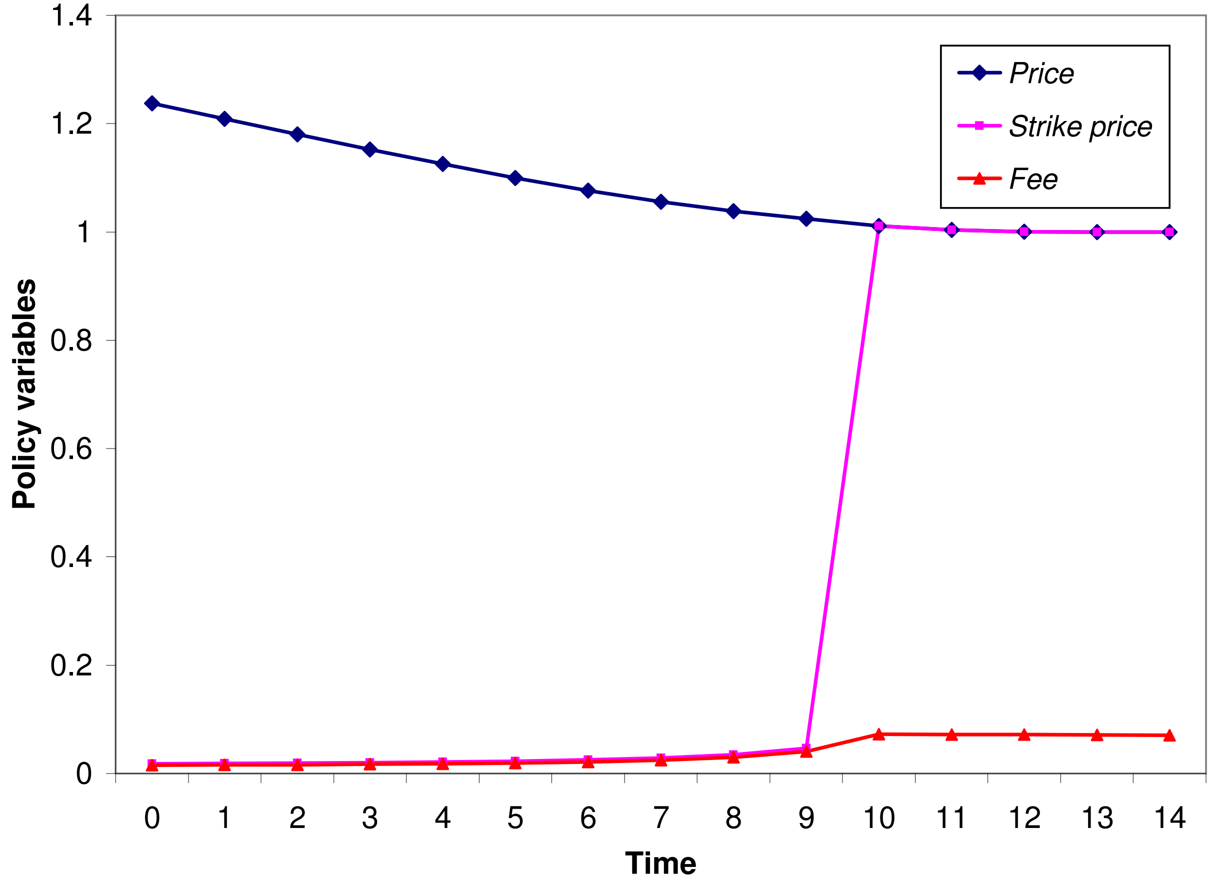
$$v(p,\kappa,\phi,t) = \left(\frac{\kappa}{\kappa+\delta\phi}\right)^{\beta}(1-\phi)^{\gamma}\left(1-\frac{t}{T}\right)^{\phi/\rho}$$

$$v_0(0) = 1 \quad \text{and } \alpha,\ \beta,\ \gamma,\ \delta,\ \rho > 0$$

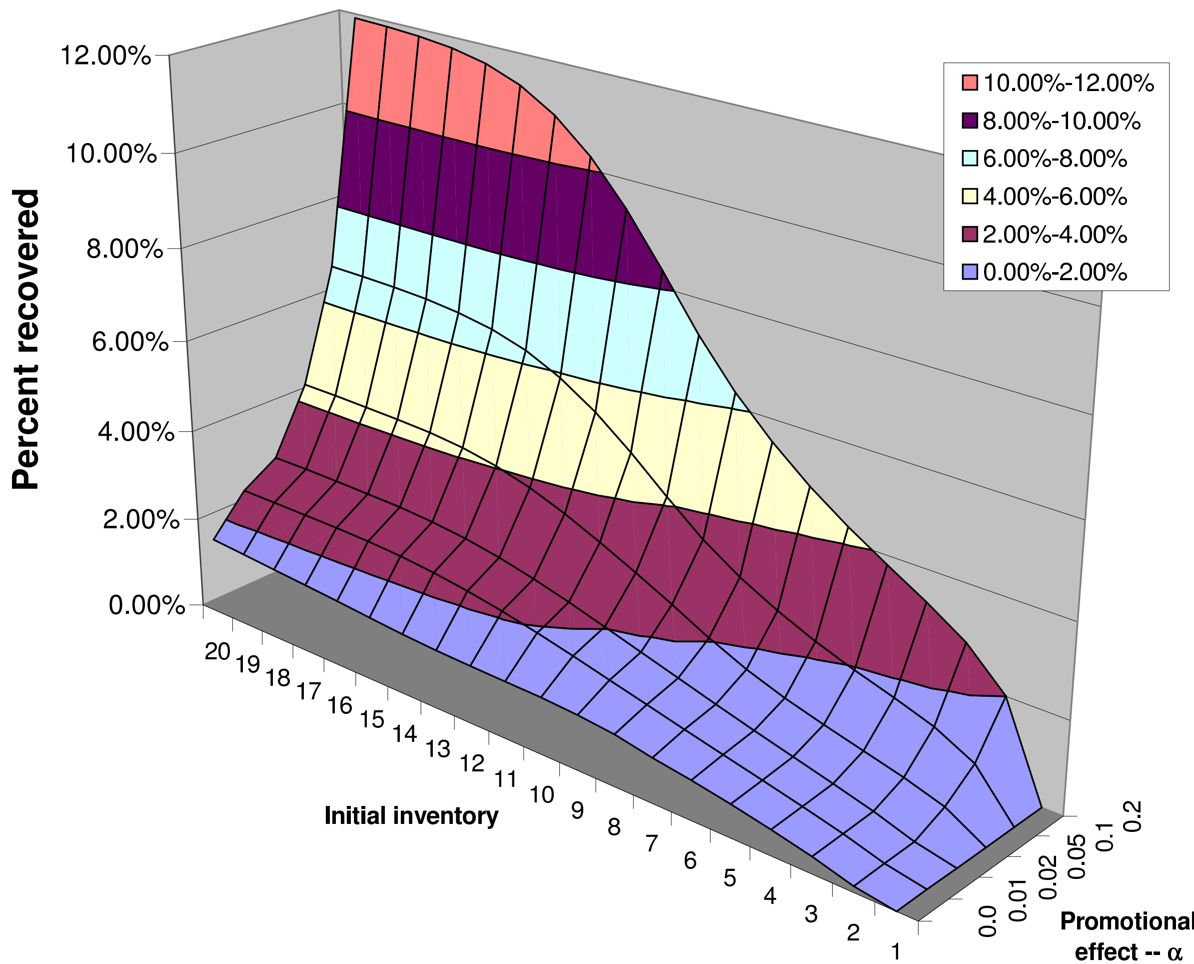
Percentage of additional revenue recovered by discrete three-item model with price guarantee: 15 time steps

	Expected number of customers		
Promotional effect α	1.5	3	6
0.00	0.8%	0.9%	0.8%
0.01	2.5%	2.3%	2.0%
0.02	2.9%	2.7%	2.4%
0.05	4.2%	4.0%	3.5%
0.10	6.6%	6.2%	5.3%
0.20	11.7%	10.9%	9.2%
0.50	29.6%	27.2%	22.2%

Discrete Model: policy variables as functions of time before first sale (15 time steps, 3 items, average of 6 customers, and no promotional effect)



Lower bound heuristic: percentage of additional revenue recovered on 50 time steps and with up to 20 items in inventory



Conclusions

- price guarantees may help manage demand in revenue management, with a potential to increase sales by reducing uncertainty of customers about future price
- exact model is not computationally tractable for larger problems
- lower bound heuristic produces pricing policies which can recover significant additional revenues