A Quantitative Analysis of Tax Enforcement and Optimal Monetary Policy

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Abstract

This paper explores the interaction and consequences of tax enforcement policies for monetary policy. Agents may evade taxes by working in an informal sector, but they are detected with positive probability. Workers are rewarded with government sponsored benefits which are proportional to formal (taxed) work. We show that when collecting taxes is costly, the optimal inflation rate is positive and inflation becomes a second best tax. Using U.S. data, we compute the quantitative effect on inflation and interest rate of evasion penalties and government provided benefits to workers in the formal sector for OECD countries. Because data are available on the amount spent to collect taxes from the informal sector, we also provide an upper bound estimate for OECD countries. This model casts doubt on the desirability and sustainability of the recent convergence in interest rates and inflation in OECD countries. Eurozone countries have coordinated their monetary policies, but they lack of fiscal policy coordination. The model clearly indicates that if countries have different tax enforcement policies and enforcement spending, then the optimal interest rate and inflation will be different.

Keywords: Optimal Interest Rate; Tax Enforcement; Informal Economy; Tax Evasion.

JEL Classification: D73; E26; E43; H26; K42.

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1 Introduction

This paper explores the quantitative interactions and consequences of tax enforcement policies for monetary policy. The model has a complete but costly set of tax instruments. Agents may evade taxes, but they risk being detected with positive probability. Furthermore, while they may be penalized for evading taxes, they are rewarded with government sponsored benefits which are proportional to formal, reported and taxed work. We show that when tax enforcement is costly, the optimal inflation rate is positive. We compute the quantitative effect on inflation and interest rates of evasion penalties and benefits. Optimal inflation and interest rates differ across countries and differences in tax enforcement policies can account for the results, given the same enforcement spending for all countries. We show that the effects of the benefits are stronger, suggesting that reward policies are more desirable than deterrence policies.

In order to analyze quantitatively the effects of tax enforcement policies, we calibrate our model for the United States. Country specific tax enforcement policies are then used in the calibrated model, keeping all other parameters fixed. We show how the optimal inflation and interest rates are affected by tax enforcement policies (tax rate, evasion penalty and benefit), given a enforcement spending technology. My analysis leads to two important lessons for policymaking, namely (i) fiscal policies affect inflation and interest rates because they alter the distortion arising from labor tax evasion and avoidance; and (ii) structural conditions and institutions constrain the set of fiscal policy tools and must be considered in the formulation of optimal monetary policy. Monetary policies that keep nominal interest rates and inflation low are constrained optimal only when these structural distortions are reduced.

Figure 1 indicates that, since 1990, long term interest rates and inflation in Eurozone countries and the United States have dropped sharply and converged to similar levels, a pattern observed for other OECD countries. Monetary policy is similar, yet the debt panel indicates that fiscal policies vary considerably. The size of the informal sector also varies positively with the percentage of debt, suggesting that countries which have
difficulty collecting taxes or face amounts of large tax evasion have high debt. In the Appendix, we document the differences in income taxes, benefits and evasion penalties.

The period 2000 to 2005 is of particular interest. The European monetary union began in 1999 and to ensure a sufficient degree of monetary convergence among member countries, the 1992 Maastricht Treaty established several criteria. A country could join the Euro area only if it had an inflation rate no higher than 1.5 percent points above the three best-performing member states and a nominal long-term interest rate no more than 2 percent points above the three best-performing member states. These criteria were also reinforced with further rules that annual budget deficits could be no more than 3 percent of the GDP and accumulated public debt no greater that 60 percent of the GDP. As Figure 1 illustrates, interest rate and inflation differentials across the Euro area countries diminished substantially. In the early 1990s, the range between high and low rates across European countries was 10 percentage points or more. By 1999, the range from the highest to lowest inflation rate had dropped to 2 – 4 percent. On the other hand, the fiscal criteria were not strictly achieved by all countries, for instance, Belgium, Greece and Italy (Lane, 2006).

This paper analyzes the following question: Is this recent convergence in inflation and interest rates optimal when markedly different fiscal policies are followed? We answer this question by conducting a quantitative analysis of the relationship between inflation and interest rates with fiscal policy (i.e., the tax enforcement spending, benefits and penalties) in OECD countries, except the Transition Economies, Iceland and Luxembourg. We focus on the case where government detection is imperfect and penalties are bounded (i.e., we rule out perfect and zero detection probabilities and infinite penalty schemes). This study will cast doubt upon the desirability of monetary policy convergence for these countries. Tax enforcement policies vary considerably across OECD and Euro area countries and play an important role in the determination of optimal monetary policy.

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1 Previous studies have analyzed the extreme cases. Either the probability of detecting a tax evader is equal to one and the agent must pay tax on “informal” income (Jones, Manuelli and Rossi, 1997) or the government cannot detect a tax evader (informal worker), which implies a detection probability of zero.
We find that when tax enforcement is problematic, deviations from the Friedman rule are optimal (the optimal nominal interest rate is not zero). Indeed, we show that the optimal monetary policy differs from the monetary policy observed in Figure 1, in some cases markedly. Our results raise questions regarding the desirability of monetary policy coordination when fiscal and tax enforcement policies are not coordinated. The intuition for this result is the following. An inefficient tax system, one characterized by tax avoidance and tax evasion, constrains the revenue sources of the government\(^2\). Inflation then becomes a “second best” tax due to the unavailability of alternative taxes. Penalties and benefits give the government additional instruments of enforcement. The quantitative exercise identifies a concern raised by Sargent and Wallace (1981) regarding the need for coordination of monetary and fiscal policies. Eurozone countries have coordinated their monetary policies through the adoption of a single currency and a central bank, but

they have not yet coordinated their fiscal policies. The model clearly indicates that if countries have different tax enforcement policies and enforcement spending, then the optimal interest rate and inflation will be different.

The model is built on Chari, Christiano and Kehoe (1996), with three main differences. The set of tax instruments is complete but taxes are costly to collect. Agents try to evade taxes by underreporting labor income to the tax authorities or working in the informal sector, but are detected with positive probability and fined. This paper uses the terms tax evasion and informal activities interchangeably. The government has carrot and stick policies which affect agents’ decisions, rewarding work in the formal sector and penalizing tax evasion. Without labor tax evasion and costly tax collection, money would not be taxed and a zero inflation tax would be optimal, the Friedman rule (Friedman, 1969). There is extensive discussion in the literature exploring conditions for the optimality of the Friedman rule and reasons for deviating from it\textsuperscript{3}. For example, previous studies have explained deviations from the Friedman Rule when agents can evade taxes by working in the informal sector where they cannot be detected, see Nicolini (1998), Cavalcanti and Villamil (2003), and Koreshkova (2006). In a model with an informal sector, Yesin (2004) evaluates the effect of costly income tax collection but the costs apply only to the formal sector. The key distinction between our model and previous studies is to recognize that the government has additional tools to deal with tax evasion and informal activities, costly tax collection and tax enforcement, a penalty to fine evaders, and a benefit paid only to workers in the formal sector which rewards tax compliance. The Friedman rule still fails, as would be expected, but optimal deviations are smaller, even for countries with large informal sectors. In a recent article, Chari and Kehoe (2006) emphasize that the robust finding is not that nominal interest rates should be literally zero, but that nominal interest rates and inflation rates should be low.

This paper is also related to a great deal of literature on tax evasion and enforcement. Becker (1968) argues that a government should set the penalty for evasion high and the costly monitoring probability low to maximize the \textit{ex ante} utility of a representative

\textsuperscript{3}See for instance Aizenman (1983), Vegh (1989) and De Fiore (2000).
agent. This policy deters evasion at minimal cost. The tax law literature also advocates this policy because it economizes on enforcement costs without sacrificing deterrence and raises revenue, according to Polinsky and Shavell (2005). The policymaker can save resources and achieve the same level of compliance by increasing penalties imposed only on dishonest taxpayers, and reducing the probability of detection.

In the original Allingham and Sandmo (1972) model, high penalties and a high monitoring probability discourage cheating. However, Slemrod and Yitzhaki (2002) argue that this kind of model ignores the possibility of a corrupt tax administrator who abuses the system or severely punishes agents who commit an honest mistake. The harsher the penalty scheme, the greater the damage. Agents may enter the underground sector to find shelter from such government-induced distortion as excessive taxes, regulation, weak legal systems or corruption, e.g., see Giles and Tedds (2002) or Friedman et al. (2000). Weaker legal systems and more corrupt environments are strongly correlated with the size of the informal economy, as will be the case in our model.

The paper is organized as follows: Section 2 presents the economy and the structure of the model. We state the Ramsey problem and consider the optimality of the Friedman rule in the presence of tax evasion and costly tax collection. In Section 3, we calibrate the model and solve it numerically. The optimal monetary policy is quantified in Section 4, under statutory penalties. Section 5 presents policy implications and Section 7 offers concluding comments.

2 Model

2.1 The Environment

The economy is populated by a large number of identical, infinitely lived consumers in discrete time. There are no information problems and markets are complete. Agents have one unit of labor that is used for leisure or to produce a single consumption good. There is no aggregate uncertainty.

The representative agent has preferences represented by a discounted lifetime utility
function
\[ \sum_{t=0}^{\infty} \beta^t U(c_{1t}, c_{2t}, h_t) \]
where \( c_{1t} \) is consumption of a cash good, \( c_{2t} \) is consumption of a credit good and \( h_t \) is leisure in period \( t \). Cash and credit goods are distinguished solely by the means of payment, which will be determined by a cash-in-advance constraint. The distinction is relevant because informal payments usually occur in cash. Let \( \beta \in (0, 1) \) be the discount factor and \( U(\cdot) \) be a strictly concave, twice continuously differentiable function, separable in consumption and leisure, that satisfies the INADA conditions.

The agent is endowed with one unit of time which can be spent on formal work (\( l_{1t}^F \)), informal work (\( l_{1t}^I \)) or leisure (\( h_t \)), i.e., \( l_{1t}^F + l_{1t}^I + h_t = 1 \).

Labor services are the only factor of production, where \( l_{1t}^F \) and \( l_{1t}^I \) are the labor supplied by the representative agent to the formal and informal sectors, respectively. The final consumption good is a composite produced by both sectors, where total output is given by the constant returns to scale production function \( F(l_{1t}^F, l_{1t}^I) \). Competitive pricing ensures that workers are paid their marginal products, \( w_{1t}^F = F_{l_{1t}^F}(t) \) and \( w_{1t}^I = F_{l_{1t}^I}(t) \), where \( F_{l_{1t}^F}(t) \) and \( F_{l_{1t}^I}(t) \) denote the marginal products of formal and informal labor and \( w_{1t}^F \) and \( w_{1t}^I \) are the wage rates for formal and informal labor, respectively.

There is a complete set of tax instruments. The government can tax all goods and services, both formal and informal labor income, but the tax on informal income is an imperfect instrument. The three distinct policy variables are the following: (i) tax rate on formal labor income (\( \tau_{1t}^F \)), (ii) revenue neutral benefit (\( b_t \)) and (iii) penalty for failing to pay the formal tax rate (\( \lambda_t \)).

We assume that the government can observe and tax the transactions occurring in the formal sector, and that the level of government spending \( g \) is fixed and determined exogenously. The tax rate \( \tau_{1t}^F \) is proportional to formal labor income. Benefit \( b_t \), paid by the government, is proportional to hours worked in the formal sector and is meant to capture the idea that agents working in the formal sector receive benefits from government services such as social security, child care and unemployment benefits. It also provides
a way to reward agents for truthful reports. Education, roads and security services, for instance, are not considered since the government cannot exclude informal workers. Transfers and taxes are revenue neutral.

Agents attempt to evade formal taxes by working in the informal sector, but there is a probability of being caught. Given an exogenous detection probability \( \pi_t \), the government punishes tax evaders and informal workers by imposing a penalty \( \lambda_t \).\(^4\) We assume that \( \lambda_t \) is proportional to the amount of tax evaded and is bound. We focus on the interesting case, \( \lambda_t \in (1, \lambda_{M}) \), where \( \lambda_{M} \) is the exogenous maximal penalty level that the government can impose; when \( \lambda_t = 1 \) no penalty is imposed. The upper bound on the penalty is motivated by the fact that countries usually adopt a range for this type of penalty. We abstract from other types of penalties such as imprisonment. We assume that the probability of detection is exogenously given and independent of government enforcement expenditures. Following Lemieux et al. (1994) and Fugazza and Jacques (2003), we assume that the tax rate applied to a person who evades taxes or works underground is \( \lambda_t \tau_t^F \). The expected informal tax rate (audit rate) on evaded income is thus given by \( \tau_t^I = \pi_t \lambda_t \tau_t^F \).

### 2.2 The Consumer

Given a probability of detection \( \pi_t \), the consumer’s problem is to maximize expected discounted lifetime utility

\[
\max \{c_1t, c_2t, h_t, M_{t+1}, B_{t+1}\} E \left[ \sum_{t=0}^{\infty} \beta^t U(c_1t, c_2t, h_t) \right]
\]  

subject to

\[
M_{t+1} + B_{t+1} = R_t B_t + M_t - p_t c_{1t} - p_t c_{2t} + p_t (1 - \tau_t^F + b_t) w_t^F \tau_t^F + p_t (1 - \tau_t^I) w_t^I \tau_t^I
\]  

\(^4\)The magnitude of penalties is the object of debate in the enforcement literature. If the government is free to choose the penalties, Becker (1968), Chander and Wilde (1998), Krasa and Villamil (2000), Polinsky and Shavell (2005), among others, have shown that (extremely) severe penalties are optimal. However, less-than-maximum fines can be optimal when enforcement is uncertain (see Polinsky and Shavell (2005) for a survey) or social norms impose economic restrictions on the penalty function (see Marhuenda and Ortuno-Ortin (1997)).
\[ p_t c_{1t} \leq M_t \]  \hspace{1cm} (3)

\[ l_t^F + l_t^I + h_t = 1 \]  \hspace{1cm} (4)

\[ -B \leq \frac{B_t}{p_t} \leq B \]  \hspace{1cm} (5)

\[ M_t, c_{1t}, c_{2t}, h_t, l_t^I, l_t^F \geq 0 \]  \hspace{1cm} (6)

where \( M_t \) is money holdings, \( B_t \) nominal bonds, \( R_t \) the interest rate paid on bonds, \( p_t \) the price level in the economy, \( \tau_t^F \) the tax rate on formal labor income, \( b_t \) the transfer and \( \tau_t^I = \pi_t \lambda_t \tau_t^F \) is the expected tax paid on labor in the informal sector given penalty \( \lambda_t \) and detection probability \( \pi_t \).

The agent receives a return on bonds acquired previously, faces a cash-in-advance constraint, consumes, and then acquires new bonds and new cash for the next period with the remaining income. Constraint (2) is the representative agent’s budget constraint. The left-hand-side is the nominal value of the assets held at the beginning of the next period. The first term on the right-hand-side is the value of nominal debt bought in the current period. The next two terms are the agent’s unspent cash. The fourth term is the payment for credit goods. The last two terms are the formal and informal net labor income, respectively. The earnings in the formal sector are deterministic and a linear function of hours of work and the net earnings in this sector is given by \( (1 - \tau_t^F + b_t) w_t^F l_t^F \).

Regarding the agent’s expected informal labor income, notice that with a given probability \( \pi_t \) the agent is caught evading tax and must pay a penalty \( \lambda_t \) proportional to the amount of the tax evaded. The agent’s informal income is thus stochastic\(^5\). The agent gets \( (w_t l_t^I - \lambda_t \tau_t^F w_t^I l_t^I) \) with probability \( \pi_t \) and \( (w_t l_t^I - 0) \) with probability \( (1 - \pi_t) \). Hence, the agent’s expected income, if caught, is \( \pi_t \left( 1 - \lambda_t \tau_t^F \right) w_t^I l_t^I \) and \( (1 - \pi_t) w_t^I l_t^I \), otherwise.

The punishment \( \lambda_t \tau_t^F w_t^I l_t^I \) is linear in the amount of tax the agent is trying to evade, hence expected informal income is \( \left( 1 - \tau_t^I \right) w_t^I l_t^I \), where \( \tau_t^I = \pi_t \lambda_t \tau_t^F \).

Purchase of cash goods must satisfy the cash-in-advance constraint (3). Money is introduced and withdrawn through open market operations. The total time spent on

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\(^5\)Because markets are complete, the agent can hedge this risk.
formal, informal work and leisure is 1, equation (4). Ponzi schemes are ruled out by constraint (5).

2.3 The Government

The government finances an exogenously given expenditure \((g_t)\) through printing new money, issuing new bonds, collecting labor income taxes from formal workers, enforcing the tax code and imposing penalties on informal workers. However, it incurs costs to collect formal and informal taxes. Government spending to collect formal taxes \((s^F_t)\) is assumed to be small and constant\(^6\). We assume that the tax enforcement spending \((s^I_t)\), or cost to collect informal taxes, is proportional to expected enforcement tax revenue \((\tau^I_t w^I_t l^I_t)\), given by

\[
s^I_t = \phi \tau^I_t w^I_t l^I_t
\]

where \(\phi\) is interpreted as the percentage of expected enforcement revenue spent to collect taxes from the informal sector and \(\tau^I_t = \pi_t \lambda_t \tau^F_t\). Given a probability of detection \(\pi_t\), the government’s period budget constraint is

\[
p_t g_t + p_t s^F_t + p_t s^I_t + R_t B_t = B_{t+1} + M_{t+1} - M_t + p_t (\tau^F_t - b_t) w^F_t l^F_t + p_t \tau^I_t w^I_t l^I_t
\]

The left side of equation (8) contains government expenditures \((g_t)\), spending to collect formal and informal taxes \((s^F_t, s^I_t)\) and current period debt service. The terms on the right side are government revenues generated by asset sales, formal tax revenue net of transfers paid to formal workers and informal tax revenue, respectively.

2.4 Resource Constraint

The resource constraint in this economy is

\[
c_{1t} + c_{2t} + g_t + s^F_t + s^I_t = F(l^F_t, l^I_t)
\]

\(^6\)De Fiore (2000) and Yesin (2004) incorporate this feature in their models and show that it is not sufficient to account for deviations from the Friedman rule. Also, OECD (2004) data support this assumption.
2.5 Competitive Equilibrium

**Definition 1** A Competitive Equilibrium is a sequence of allocations \( x = \{x_t\}_{t=0}^\infty \) where \( x_t = (c_{1t}, c_{2t}, l^F_t, l^I_t, M_t, B_t) \), a sequence of prices \( p = \{p_t, w^I_t, w^F_t\}_{t=0}^\infty \), a sequence of government policies \( \tau = \{g_t, \tau^F_t, b_t, \lambda_t\}_{t=0}^\infty \), a sequence of government bond prices \( R = \{R_t\}_{t=0}^\infty \) and a given probability of detection \( \pi_t \), such that (i) given \((p, \tau, R)\), the allocation sequence \( x = \{x_t\}_{t=0}^\infty \) solves the representative agent’s utility maximization problem and (ii) given \( \{g_t, \tau^F_t, \lambda_t, b_t\}_{t=0}^\infty \) the resource constraint is satisfied each period.

Let \( \beta^t \eta_t \) and \( \beta^t \mu_t \) be the Lagrange multipliers for consumer budget constraint (2) and for CIA constraint (3), respectively. The first order conditions with respect to \( c_{1t}, c_{2t}, l^F_t, l^I_t, B_{t+1}, M_{t+1} \) are the following:

\[
U_{c_1}(t) = p_t (\eta_t + \mu_t) \tag{10}
\]

\[
U_{c_2}(t) = p_t \eta_t \tag{11}
\]

\[
U_h(t) = p_t \eta_t \left( 1 - \tau^F_t + b_t \right) w^F_t \tag{12}
\]

\[
U_h(t) = p_t \eta_t \left( 1 - \pi_t \lambda_t \tau^F_t \right) w^I_t \tag{13}
\]

\[
\eta_t = \beta \eta_{t+1} R_{t+1} \tag{14}
\]

\[
\eta_t = \beta (\eta_{t+1} + \mu_{t+1}) \tag{15}
\]

and the equilibrium conditions can be represented as

\[
\tau^F_t - b_t = 1 - \left( \frac{U_h(t)}{U_{c_2}(t)} \frac{1}{w^F_t} \right) \tag{16}
\]

\[
\tau^I_t = \left( 1 - \frac{U_h(t)}{U_{c_2}(t)} \frac{1}{w^I_t} \right) \tag{17}
\]

\[
R_{t+1} = \frac{U_{c_2}(t)}{\beta U_{c_2}(t+1)} (1 + \Pi_{t+1}) \tag{18}
\]
\[ \Pi_{t+1} = \beta \frac{U_{c_1}(t+1)}{U_{c_2}(t)} - 1 \]  

(19)

where \( \Pi_{t+1} = \frac{p_{t+1} - p_t}{p_t} \) is the inflation rate.

Given that the gross nominal return on money is equal to one, in any equilibrium \( R_{t+1} \geq 1 \) because otherwise the consumer could make infinite profits by buying money and selling bonds. From agent equilibrium conditions (18) and (19), the condition \( U_{c_1}(t) \geq U_{c_2}(t) \) must hold in any equilibrium.

### 2.6 The Ramsey Problem

The social planner’s goal is to maximize a representative agent’s utility subject to raising an exogenously determined amount of revenue for the government, taking into account the equilibrium reactions by consumers and firms to the distortionary tax system. A Ramsey problem characterizes the set of allocations that can be implemented as a competitive equilibrium with distorting taxes subject to a resource and implementability constraint.

**Proposition 1 (Ramsey Allocation)** For a given probability of detection \( \pi_t \), the consumption and labor allocations in the Ramsey equilibrium solve the Ramsey problem

\[
\max_{\{c_{1t}, c_{2t}, h_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t U(c_{1t}, c_{2t}, h_t)
\]

subject to

\[
c_{1t} + c_{2t} + g_t + s^F_t + s^I_t = F(t^F_t, t^I_t)
\]

(21)

\[ U_{c_1}(t) \geq U_{c_2}(t) \]

(22)

\[ \sum_{t=0}^{\infty} \beta^t [c_{1t}U_{c_1}(t) + c_{2t}U_{c_2}(t) - (1 - h_t)U_h(t)] = 0 \]

(23)

**Proof.** The proof of Proposition 1 follows Chari, Christiano and Kehoe (1996) and has two parts: (i) Allocations in a competitive equilibrium must satisfy the implementability constraint (23) and resource constraint (21) and, conversely, (ii) Any allocation satisfying (21), (22), and (23) can be decentralized as a competitive equilibrium.
The Ramsey planner’s problem is to maximize the discounted present value of utility (1) subject to resource constraint (9), government budget constraint (8), and the constraints imposed by household optimization. In the first part of the proof, the planner chooses directly the optimal allocations. Constraint (23) is the consumer budget constraint with both taxes and prices substituted out by the first order conditions from the agent’s utility maximization problem. (23) is the implementability constraint which restricts the set of allocations that can be implemented as a competitive equilibrium with distorting taxes. To derive the implementability constraint, multiply the expected agent budget constraints by their Lagrange multipliers (βtηt) for each period, sum over t, and use the first order conditions of the agent’s optimization problem (16) – (19). This yields the following

\[ \sum_{t=0}^{\infty} \beta^t [c_{1t}U_{c1}(t) + c_{2t}U_{c2}(t) - (1 - h_t)U_h(t)] = \eta_0 R_0(M_0 + B_0) \]

We assume that the government is constrained in its policy choices in the first period and sets the initial stock of nominal assets, \(M_0 + B_0\), to zero. If the initial stock is either negative or positive, welfare is maximized by setting the initial price level to either negative infinity or infinity, respectively. Assumption \(M_0 + B_0 = 0\) does not affect the results and simplifies the calculations. Hence we obtain the implementability constraint (23)

\[ \sum_{t=0}^{\infty} \beta^t [c_{1t}U_{c1}(t) + c_{2t}U_{c2}(t) - (1 - h_t)U_h(t)] = 0 \]

The resource constraint for this economy (21) can be represented as

\[ c_{1t} + c_{2t} + g_t + s_t^F + s_t^I = F(l_t^F, l_t^I) \]

Given that the gross nominal return on money is equal to one, in any equilibrium \(R_t \geq 1\) because otherwise the consumer could make infinite profits by buying money and selling bonds. According to the agent’s equilibrium conditions, constraint (22), \(U_{c1}(t) \geq U_{c2}(t)\), must hold in any equilibrium.
We can construct prices and government policies such that (21), (22), and (23) characterize the competitive equilibrium allocations. These prices and policies can be characterized by conditions (16) – (19) from the agent’s optimization problem. We also use the firm’s first-order conditions, \( w^F_t = F_{Ft}(t) \) and \( w^I_t = F_{It}(t) \), where \( F_{Ft}(t) \) and \( F_{It}(t) \) denote the marginal products of formal and informal labor, and \( w^F_t \) and \( w^I_t \) are the wage rates for formal and informal labor, respectively.

Define the formal (net) tax rate and the expected penalty in each period \( t \) to be

\[
\tau^F_t - b_t = 1 - \left( \frac{U_h(t)}{U_c(t)} \frac{1}{w^F_t} \right) \quad \text{and} \quad \pi_t \lambda_t = \frac{1}{\tau^F_t} \left( 1 - \frac{U_h(t)}{U_c(t)} \frac{1}{w^I_t} \right).
\]

Also construct the interest rate as

\[
R_{t+1} = \frac{U_c(t)}{U_c(t)}.
\]

Given the assumptions on the utility function, the first order conditions are necessary and sufficient for agent and firm maximization. The price level is indeterminate and all variables in the economy can be defined in real terms. Real money holdings will be equal to the consumption of the cash good, \( \frac{M}{p_t} = c_{1t} \). Real bond holdings can be obtained following a similar method used to construct the implementability constraint. This completes the characterization of the competitive equilibrium.

The solution to the Ramsey problem is an allocation that maximizes social welfare, subject to the restriction that it can be decentralized as a competitive equilibrium with taxes. Equation (21) is the resource constraint taking into account the consumer problem first order conditions. Money earns a gross nominal return of 1 and, from the consumer problem, in equilibrium \( R \geq 1 \). Thus, in any equilibrium constraint (22) must hold. The implementability constraint, equation (23), is a constraint on the set of allocations that can be implemented as a competitive equilibrium with distorting taxes.

### 2.7 The Optimality of the Friedman Rule

In an economy without distortions, the optimal monetary policy is characterized by a nominal interest rate equal to zero, according to Friedman (1969). There is an extensive discussion in the literature exploring conditions for the optimality of the Friedman rule and reasons for deviation from it. Aizenman (1983) and Vegh (1989), for example, argue that the Friedman rule is not optimal in economies with a costly tax system. De Fiore
(2000) considers the importance of (formal tax) collection costs in generating optimal deviations from the Friedman rule, however the costs do not justify the substantial deviations from Friedman’s prescription in her numerical exercise. The Friedman rule also fails when agents can evade taxes by working in the informal sector, see Nicolini (1998) and Cavalcanti and Villamil (2003). Finally, in a model with an informal sector Yesin (2004) evaluates the effect of costly income tax collection only in the formal sector.

In our economy, the government can tax labor in both sectors (or equivalently both consumption goods), but agents attempt to evade taxes by working underground. With a given positive probability, they are caught. An exogenous expected informal penalty \( \pi_t \lambda_t \) is an imperfect instrument for reducing the wedge between the marginal rate of substitution of consumption and informal labor and the marginal rate of transformation. Since tax evasion and informal activities are heavily carried out in cash and imperfectly observed by the tax authorities it is reasonable to assume that the marginal cost of taxing cash goods payments is higher that the cost of taxing credit goods, i.e. \( s^I_{c_1} > s^I_{c_2} \).

Moreover, purchases of credit goods are assumed to be perfectly monitored by the tax authorities, because the agent issues nominal claims, which are settled in the security market, to purchase credit goods.

**Proposition 2** Assume that, given a probability of detection \( \pi_t \in (0,1] \) an informal worker is detected and must pay the informal tax rate on evaded income \( (r^I_t > 0) \). If tax enforcement spending is positive, \( s^I_t > 0 \), then the optimal interest rate is positive (i.e. the Friedman rule is not the optimal monetary policy).

**Proof.** Consider the Ramsey problem, as follows:

\[
\max_{\{c_{1t}, c_{2t}, h_t\}_{t=0}^\infty} \sum_{t=0}^{\infty} \beta^t U(c_{1t}, c_{2t}, h_t)
\]

subject to the resource constraint (21), market equilibrium condition (22) and the implementability constraint (23). Let \( \xi_t \) and \( \beta^t \gamma_t \) be the Lagrange multipliers for (21) and (23), respectively. The first order conditions with respect to \( c_{1t} \) and \( c_{2t} \) are, respectively
3.2.1 and 3.2.2,

\[(1 + \xi)U_1(t) + \xi[U_{11}(t)c_{1t} + U_{21}(t)c_{2t} - U_{31}(t)(1 - h_t)] = \gamma_t [1 + s^{I}_{c_1}] \quad \text{(3.2.1)}\]

\[(1 + \xi)U_2(t) + \xi[U_{12}(t)c_{1t} + U_{22}(t)c_{2t} - U_{32}(t)(1 - h_t)] = \gamma_t [1 + s^{I}_{c_2}] \quad \text{(3.2.2)}\]

where \(U_i(t)\) denotes \(U_i(c_{1t}, c_{2t}, h_t)\), \(i = 1, 2, 3\), \(M_1 = \left(\frac{U_{31}U_{22} - U_{32}U_{21}}{(U_{22})^2}\right)\) and \(M_2 = \left(\frac{U_{32}U_{22} - U_{31}U_{21}}{(U_{22})^2}\right)\), and \(s^{I}_{c_1} = \frac{\partial s^I}{\partial c_1}\) and \(s^{I}_{c_2} = \frac{\partial s^I}{\partial c_2}\). Consider utility functions of the form \(U(c_{1}, c_{2}, h) = V(w(c_{1}, c_{2}), h)\), where \(w\) is homothetic. A utility function of this form also satisfies the following:

\[
\frac{U_{11}(t)c_{1t} + U_{21}(t)c_{2t}}{U_1(t)} = \frac{U_{12}(t)c_{1t} + U_{22}(t)c_{2t}}{U_2(t)}
\]

\(\text{(3.2.3)}\)

Dividing 3.2.1 and 3.2.2 by \(U_1(t)\) and \(U_2(t)\), respectively and noting that \(\frac{U_i}{U_j} = \frac{V_{i}}{V_{j}}\) for \(i = 1, 2\) we have that

\[
(1 + \xi) + \xi \left[\frac{U_{11}(t)c_{1t} + U_{21}(t)c_{2t}}{U_1(t)} - \frac{V_{12}}{V_1} (1 - h_t)\right] = \frac{\gamma_t [1 + s^{I}_{c_1}]}{U_1(t)} \quad \text{(3.2.4)}
\]

\[
(1 + \xi) + \xi \left[\frac{U_{12}(t)c_{1t} + U_{22}(t)c_{2t}}{U_2(t)} - \frac{V_{12}}{V_1} (1 - h_t)\right] = \frac{\gamma_t [1 + s^{I}_{c_2}]}{U_2(t)} \quad \text{(3.2.5)}
\]

Using (3.2.3), we have that the left side of (3.2.4) and (3.2.5) have the same value for \(i = 1\) and for \(i = 2\). This implies that \(\frac{\gamma_t [1 + s^{I}_{c_1}]}{U_1(t)} = \frac{\gamma_t [1 + s^{I}_{c_2}]}{U_2(t)}\) and, consequently,

\[
\frac{U_1(t)}{U_2(t)} = \frac{1 + s^{I}_{c_1}}{1 + s^{I}_{c_2}} \quad \text{(3.2.6)}
\]

From the consumer’s first order condition \(\frac{U_1(t)}{U_2(t)} = R_t\) and \(s^{I}_{c_1} > s^{I}_{c_2}\), We have that \(R_t > 1\), i.e., the optimal interest rate is positive. Hence, in the presence of positive collection cost, the Friedman rule is not the optimal monetary policy. This completes the proof of Proposition 2. ■

In a tax system with costly collection, the optimal inflation tax is positive, i.e., the Friedman rule is not optimal. When the informal sector cannot be perfectly observed and money is used for transactions in this sector, the optimal policy is to set a positive
inflation tax, in addition to two other distortionary tax instruments, namely a positive formal income tax ($\tau^F_t > 0$) and a positive evasion penalty ($\lambda_t > 0$). A positive income tax increases the distortion in the agents’ consumption-leisure choice. A positive inflation tax affects the agents’ decisions to hold real balances. Similarly, a positive evasion penalty and benefit affect the worker’s time allocation in the informal sector. The government spends resources to collect informal labor income taxes; this enforcement spending is a dead weight loss. However, higher evasion penalties deter agents from working in the informal sector, reducing the distortion on the second margin.

3 Quantitative Analysis

3.1 Measurement: The United States as the Baseline Economy

3.1.1 Functional Forms and the Ramsey Problem

Following Chari, Christiano and Kehoe (1991, 1996) we assume a CES utility function given by the following:

$$U(c_{1t}, c_{2t}, h_t) = (1 - \eta)^\frac{1}{\nu} \log [(1 - \sigma)(c_{1t})^\nu + \sigma(c_{2t})^\nu] + \eta h_t$$  \hspace{1cm} (24)

where $c_{1t}$, $c_{2t}$ and $h_t$ denote the cash good, the credit good and leisure in period $t$, respectively. The preference parameters $\eta$, $\sigma$ and $\nu$ represent the work-leisure time allocation, the cash-credit goods weight and the coefficient of risk aversion, respectively.

As in Easterly (1993), aggregate output is given by a constant returns to scale production function in the following form:

$$F(l^F_t, l^I_t) = [\alpha(l^I_t)^\rho + (1 - \alpha)(l^F_t)^\rho]^{\frac{1}{\rho}}$$  \hspace{1cm} (25)

Output is a function of the two types of labor, formal ($l^F_t$) and informal ($l^I_t$), with elasticity of substitution ($1/1 - \rho$). Technology parameter $\alpha$ denotes the percentage of informal
labor in production\textsuperscript{7}.

For an economy with utility function (24) and production function (25), the Ramsey problem is to choose consumption and labor allocations that solve the following optimization problem:

\[
\max \sum_{t=0}^{\infty} \beta^t \left[ (1 - \eta) \frac{1}{v} \log [(1 - \sigma)(c_{1t})^v + \sigma(c_{2t})^v] + \eta(1 - l_t^F - l_t^I) \right]
\]

subject to

\[
c_{1t} + c_{2t} + g_t + s_t^F + \phi \left( F(t) - \frac{\eta}{(1 - \eta)\sigma} \frac{(1 - \sigma)(c_{1t})^v + \sigma(c_{2t})^v}{(1 - l_t^F - l_t^I)(c_{2t})^{v-1}} \right) l_t^I = F(t^F, l_t^I)
\]

\[
(1 - \sigma)(c_{1t})^{v-1} \geq \sigma(c_{2t})^{v-1}
\]

\[
\sum_{t=0}^{\infty} \beta^t \left[ (1 - \eta) + \eta \frac{l_t^F + l_t^I}{1 - l_t^F - l_t^I} \right] = 0
\]

\[c_{1t}, c_{2t}, l_t^F, l_t^I \geq 0\]

The expression for the optimal interest rate for this economy is obtained by the procedure described in the proof of Proposition 2, with preferences and production function given by (24) and (25), respectively.

3.1.2 Calibration Strategy

We calibrate the model for the United States, the baseline economy. Suppose that government consumption expenditures \((g_t)\) are constant and assume that the solution to the Ramsey problem converges to a steady state. We solve for the optimal steady state interest rate in terms of preference and production parameters. The parameters of the model are \(\eta, v, \sigma, \beta, \alpha, \rho\). The baseline values are summarized in Table 1.

A period is a year and we assume that discount factor \(\beta\) is 0.96. One quarter of an agent’s time is allocated to market activities (40 hours per week). In the steady state, the implementability constraint implies \(h = \eta\), thus \(\eta\) equals 0.75. For the values of \(\sigma\)

\textsuperscript{7}These functional forms for the utility and production functions are standard in quantitative studies of the Friedman rule. See for instance Cavalcanti and Villamil (2003) and Yesin (2004).
and $v$, we use 0.57 and 0.83, respectively, estimated by Chari, Christiano and Kehoe (1991) for the United States. The elasticity of substitution between formal and informal labor, $\rho$, is assumed to be 0.71, based on the estimation in Lemieux, Fortin and Frechette (1994) for Canada. Since no similar data are available for the United States, and given the similarities between the two economies, we use this parameter as a proxy for $\rho$ in the United States.

### Table 1 - Baseline Values

<table>
<thead>
<tr>
<th>Preferences: Consumer</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$ discount factor</td>
<td>0.96</td>
<td>U.S. data</td>
</tr>
<tr>
<td>$\eta$ work-leisure time allocation</td>
<td>0.75</td>
<td>Time data</td>
</tr>
<tr>
<td>$\alpha$ cash-credit goods weight</td>
<td>0.57</td>
<td>Chari et al. (1991)</td>
</tr>
<tr>
<td>$v$ risk-aversion term</td>
<td>0.83</td>
<td>Chari et al. (1991)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Technology: Firm</th>
<th></th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$ elasticity of substitution between formal and informal labor</td>
<td>0.71</td>
<td>Lemieux et al. (1994)</td>
</tr>
<tr>
<td>$\alpha$ percentage of informal labor in production</td>
<td>0.28</td>
<td>Calibrated</td>
</tr>
</tbody>
</table>

#### Government

| $\tau^F$ tax rate on formal income | 0.29 | Internal Revenue Service |
| $g$ government spending           | 0.20 | OECD                      |
| $b$ transfer to formal workers    | 0.08 | OECD (2004b)              |
| $\pi$ probability of detecting an informal worker | 0.0093 | Internal Revenue Service |
| $\lambda$ administrative penalty for tax evasion | 1.48 | Andreoni et al. (1998) |
| $s^F$ formal tax collection spending | 0.0052 | IRS, OECD (2004)         |
| $\phi$ informal tax collection spending parameter | 0.25 | IRS, Kenney (2005)       |

The formal tax rate $\tau^F_t$ is assumed to be the "all-in" marginal tax rate for employees. According to OECD (2004a), this tax rate includes personal income tax and employee social security contributions less cash benefits. We use the marginal tax rate because it

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8In section 3.5, we report sensitivity analysis for $\rho, v$ and $\sigma$. 

may influence decisions on how many hours to work, as it gives the amount of extra wage income an individual worker keeps after taxes\(^9\). For the United States, the "all-in" tax rate is 29.1 percent, which implies \( \tau_t^F = 0.291 \).

OECD (2004b) reports public social expenditures as a percent of GDP for OECD countries. Cash transfers include seven groups of benefits that are available to families as follows: (i) Unemployment benefits; (ii) Social assistance benefits; (iii) Cash housing benefits; (iv) Family benefits; (v) Childcare benefits; (vi) Lone-parent benefits; (vii) Employment-conditional benefits. For the United States, government expenditures on all these benefits is 8 percent of GDP, and we set \( b_t = 0.08 \). Benefits in our model are transfers that only workers in the formal sector are entitled to. For simplicity we assume that the cost of paying these benefits is zero. This reflects the fact that agents will self-report their eligibility to receive benefits, but may not do so to pay taxes.

To construct the tax rate on informal income (\( \tau_t^I = \pi \lambda \tau_t^F \)) we must identify values for the probability of detecting an informal worker (\( \pi \)) and the penalty for tax evasion (\( \lambda \)). In practice, public enforcement is often characterized by low probabilities of detection. According to Andreoni et al. (1998), the audit rate for individual tax returns was 1.7 percent in 1995. Data from the Internal Revenue Service show that the audit rate dropped sharply during the period 1996-2002. The percent of returns examined for fiscal years 1996-2002 are 1.67, 1.30, 0.99, 0.90, 0.50, 0.58 and 0.56, respectively. We follow Busato and Chiarini (2004) in our calibration strategy for \( \pi \). We estimate the (unconditional) mean of the ratio of the number of individual income tax returns examined to the total number of tax returns received by the Internal Revenue Service during the period of 1996-2002. For the United States, \( \pi = 0.0093 \) or 0.93 percent.

The tax rate applied to a person who evades taxes or works underground is \( \lambda \tau_t^F \), where \( \lambda > 1 \) is the penalty. Administrative penalties for understatements of tax liability are generally imposed as a percent of the additional tax payable and vary according to the seriousness of the offence. In the United States, administrative penalties are applied at

\(^9\) Average tax rates may influence the decision to enter (or exit) the labor market, as they effect how much total net income after tax changes if one decides to join (or exit) the labor market. The average "all-in" tax rate for the U.S. is 24.1 percent. The results presented in the next section are not sensitive to the choice between marginal and average tax rates.
rates of 20 and 75 percent of the portion of underpaid tax or fraud, respectively (Andreoni, Erard and Feinstein, 1998). We assume the midpoint 48 percent as our baseline value.

We choose the government expenditures parameter $g_t$ so that the size of the government corresponds to 20 percent of formal output at the resulting Ramsey equilibrium; see OECD data in the Appendix. The government incurs costs to collect formal and informal taxes $s_t^F$ and $s_t^I$. According to OECD (2004), national revenue authorities compute and publish a "cost of collection" for formal sector taxes. The ratio is computed by comparing the annual costs of administration incurred by a revenue authority, with the revenue collected over the course of a fiscal year. This can be expressed as the amount the government spends to collect 100 units of revenue. For the United States, the government spends US$ 0.52 to collect US$ 100 of formal taxes and we set $s_t^F = 0.0052$.

For the baseline economy, the United States, according to IRS Commissioner Mark Everson (Kenney, 2005), enforcement of the tax code is expected to pay for itself and the benefit/cost ratio is more than four-to-one. This means that an additional dollar spent on enforcement returns at least four dollars to the government ($\phi = 0.25$). Therefore, tax authorities spend US$ 25 to collect US$ 100 of informal taxes. In the United States, spending to collect formal taxes, $s_t^F$, can be interpreted as the cost to run the Internal Revenue Service and carry out its main tasks. Enforcement spending requires additional effort and resources.

We calibrate the production technology parameter $\alpha$. Schneider and Klinglmair (2004) estimate the size of the shadow economies for 110 countries. We use this estimate as a proxy for tax evasion in our exercise. They estimate that the size of the informal sector (% of GDP) is 8.7 percent for the United States in 2000. The production parameter $\alpha$ is chosen so that the size of the informal sector corresponds to 8.7 percent and satisfies the following expression derived from the consumer and firm first order conditions:

$$\frac{l_t^I}{l_t^F} = \left[ \frac{\alpha}{1 - \alpha} \frac{1 - \tau_t^I}{1 - \tau_t^F + b_t} \right]^{\frac{1}{1-\sigma}}$$

(27)

Before accounting for deviations from the Friedman rule in OECD countries we can
verify the implications of the model for the United States. Assuming the production technology parameter $\alpha$ and the enforcement spending parameter $\phi$ for the United States, we estimate the size of the informal sector (% GDP) and compare the results with existing data. By construction the size of the informal sector represents 8.7 of the GDP in the model. The model underpredicts the consumption/output ratio, suggesting a figure of 81.7 percent while the OECD data indicates 86.3 percent (see Appendix). Interest and inflation rates are quite similar to the observed levels for the United States (Figure 2).

3.2 Accounting for Deviations from the Friedman Rule

In this section we fix the enforcement spending parameter ($\phi$), probability of detection ($\pi$), technology parameter ($\alpha$), elasticity of substitution parameter ($\rho$) and preference parameters ($\sigma, v$) at the United States values. We change the tax enforcement policies, $\tau_{t}^{F}, b_{t}$ and $\lambda_{t}$ to the country values in OECD, year 2004. The purpose of this exercise is two-fold: (i) to determine the optimal interest rate and inflation predicted by the model and (ii) to compare the performance of the model with the data.

Penalties for tax evasion are assumed to be at their statutory level. OECD (2004) provides international comparative data on tax systems and their administration for evasion penalty values ($\lambda$). A common approach is to levy penalties for minor offences in the region of 10 – 30 percent of the tax evaded while more serious offences involving deliberate evasion are in the region of 40 – 75 percent of the tax evaded. The tightest range is imposed by Finland, followed by Japan. Countries like Greece, Italy, Austria, New Zealand and Denmark utilize not only broader ranges but also higher penalties for the tax evaded. Sweden is an exception, applying a fixed penalty (Appendix).

Figure 2 reports the optimal interest rate (first plot) and inflation (second plot) predicted by the model and compares them to the (average) interest rate and inflation observed in OECD data, respectively, for the period 2000 – 2005. In this figure, if the model predicts the rates perfectly then the points would lie on the dashed lines. The optimal interest rates predicted by the model are in general lower than the levels observed in OECD data for the period 2000 – 2005. For instance, the observed interest rate
for France is 4.5 percent, while the optimal interest rate predicted by the model is 3.4 percent, given France’s policies ($\tau_F^t$, $b_t$ and $\lambda_t$). The model performs well for Australia, Canada, Norway, Sweden and Switzerland. For Denmark, the optimal interest rate is 7.2 percent, which is higher than the observed rate. The model underpredicts the interest rates for most of the countries, e.g. Greece, Portugal and the United Kingdom, suggesting a lower optimal interest rate, given all other parameters fixed at the U.S. level.

Regarding the optimal inflation rate, the model predicts rates higher than those observed in OECD data for most countries. In Italy, the optimal inflation rate is 6.9 percent, higher than the observed rate of 2.4 percent. The model performs fairly well for Austria, France, New Zealand, Spain, the United States and the United Kingdom. Among Euro Area countries, optimal inflation and interest rates are higher than the current levels for Belgium and Italy, but lower for Greece.

As pointed out before, both inflation and interest rates have converged to one digit rates for the majority of OECD countries. However, the predicted optimal interest rate and inflation vary considerably across countries when country-specific tax enforcement policies ($\tau_F^t$, $b_t$ and $\lambda_t$) are taken into account. The results presented in Figure 2 suggest that if fiscal institutions were considered, it would be optimal for these countries to have different inflation and interest rates. For the Eurozone countries in particular, the coordination of monetary policy does not reflect, and apparently does not consider, differences in tax enforcement policies among its members.
Figure 2 - Nominal Interest Rate and Inflation (Data and Model Predictions)

When $\tau^F_t, b_t$ and $\lambda_t$ Vary Across Countries

A reasonable question at this point is why does the model not perfectly account for the interest and inflation rate data. Notice that we use country-specific tax enforcement policies $\tau^F_t, b_t$ and $\lambda_t$, but fix the enforcement spending parameter ($\phi$), probability of detection ($\pi$), technology parameter ($\alpha$), elasticity of substitution parameter ($\rho$) and preference parameters ($\sigma, \nu$) at the United States values. We check now if the fixed parameters can help better account for the data (i.e. move closer to the 45° line). Thus, in the next section we estimate the enforcement spending for each country and conduct sensitivity analysis for the other fixed parameters.

3.3 Constructed Informal Enforcement Spending Parameter $\phi$

In this section, we obtain an estimate of informal enforcement spending parameter $\phi$ for each country. We assume that the observed long term nominal interest rate on government bonds in 2005 is the optimal interest rate, which is given by the solution to the Ramsey problem. For each country parameter $\phi$ is calibrated so that the following expression is
satisfied:

\[ \phi = \Gamma (1 - R) \left[ R + (\sigma/(1 - \sigma))^{1/\nu} R^{2\nu - 1/\nu} \right]^{-1} \]  

(28)

where \( \Gamma = (1 - \eta)/(v - 1)l^I_t \), \( \eta \) is the work-leisure time allocation parameter, \( v \) is the risk aversion term and \( l^I_t \) is determined endogenously. Given common preference and technology parameters, and country-specific tax enforcement policies, (28) gives an estimate of \( \phi \) for each country.

For the baseline economy, the United States, the calibrated collection spending for the informal sector, \( \phi \), is 0.261. This means that tax authorities spend US$ 26 to collect US$ 100 of informal taxes, which is 50 times the formal tax collection cost (\( s^F_t \)). This value (\( \phi = 0.26 \)) is very similar to what the Internal Revenue Service (IRS) estimates enforcement costs to be in the United States. According to IRS Commissioner Mark Everson (Kenney, 2005), enforcement of the tax code is expected to pay for itself and the benefit/cost ratio is more than four-to-one. This means that an additional dollar spent on enforcement returns at least four dollars to the government (\( \phi = 0.25 \)).

Figure 3 shows an inverse relationship between the size of the informal sector (% GDP) and the informal collection spending parameter \( \phi \). It suggests that in countries with larger informal sectors (for instance, Belgium, Greece, Italy, Korea, Portugal and Spain) an additional dollar spent on enforcement would return relatively more than it does in the United States. For instance, in Greece, tax authorities would spend US$ 9 to collect US$ 100 of informal taxes. This may indicate that enforcement is poor (which may be the case in Greece and Italy) or efficient (Scandinavian countries) for reasons that are exogenous. Even though these countries have similar nominal interest rates and inflation (Figure 1), we estimate quite different tax enforcement spending parameters. If the current nominal interest and inflation rates were optimal for these economies, we would have expected relatively similar informal enforcement spending parameters.
Using the constructed enforcement spending parameter ($\phi$) for each country we redo the exercise to determine the optimal interest rate and inflation predicted by the model with country-specific $\tau^F_i$, $b_i$, $\lambda_i$ and $\phi$ and fixed probability of detection ($\pi$), technology parameter ($\alpha$), elasticity of substitution parameter ($\rho$) and preference parameters ($\sigma, v$) at the United States values. Note that if differences in the four country-specific policy parameters that we consider accounted for all the differences between countries, then all points should lie on the 45° line in a plot of the data and model predictions for the nominal interest rate and inflation across countries.

Figure 4 shows the optimal interest rate (first plot) and inflation (second plot) predicted by the model and compares them to the (average) interest rate and inflation observed in OECD data, respectively, for the period 2000 – 2005. The optimal interest rates are now uniformly underpredicted by the model. However, it performs better with respect to the inflation rates in OECD countries. The under prediction by the model may be because the model does not account for concern about default by some Euro countries.
(e.g. Italy, Greece, Portugal and Spain) and Korea, which was part of the Asian financial crisis in 1997 – 98. The data certainly account for this.

![Graph showing interest rate and inflation](image)

Figure 4 - Nominal Interest Rate and Inflation (Data and Model Predictions)

When \( \tau^F_t, b_t, \lambda_t \) and \( \phi \) Vary Across Countries

### 3.4 Accounting for the Size of the Informal Sector

We can also estimate the size of the informal sector (% GDP) and compare the results with existing estimates (Appendix, Table A.2). In general, when all parameters are at the U.S. level except \( \tau^F_t, b_t, \lambda_t \), the size of the informal sector (% GDP) is smaller than the numbers estimated by Schneider and Klinglmair (2004). The model performs well for Switzerland, Denmark, the United Kingdom, New Zealand, Belgium, Germany and Finland, with gaps of 1 to 5 percent of the GDP between the numbers estimated by Schneider and Klinglmair and the figures predicted by this model. We can match the size of the informal sector for Netherlands and Austria. The model performs poorly for large informal sector countries, i.e. Portugal, Greece, Korea, Mexico and Turkey. According to Schneider and Klinglmair (2004), the size of the informal sector in these countries represents more than 20 percent of the GDP, while our model suggest figures...
around 10 percent of the GDP. These results indicate that other factors, in addition to enforcement parameters, $\tau_i^E$, $b_t$, $\lambda_t$, also contribute to the size of the informal sector. Standard explanations, that we do not incorporate in our model but are not inconsistent with it, are complexity of business registration and tax laws, credit market access, labor market flexibility, etc.\(^\text{10}\)

### 3.5 Sensitivity Analysis

To derive the results presented so far, preference parameters ($\sigma, \nu$), elasticity of substitution parameter ($\rho$) and the probability of detection ($\pi$) were kept at the baseline levels. However, they may differ across countries. We observe that the share of informal labor in production ($\alpha$) increases with the elasticity of substitution parameter $\rho$, which reflects the substitutability between formal and informal labor in the production function. If it is easy to move from the formal to the informal sector, the optimal interest rate and inflation are lower. As parameter $\alpha$ increases, the agent is more visible to the tax authorities (Lemieux et al., 1998). This increases the expected informal tax revenue for the government, reducing its necessity to rely on inflation to finance its expenditures.

This result differs from previous studies that suggested that the higher $\rho$ is, the higher the interest rate and inflation will be. When the tax system is incomplete, as in Cavalcanti and Villamil (2003) and Yesin (2004), higher substitutability between formal and informal labor implies that agents will move easily from the formal to the informal sector. Since the government has no instruments to detect and tax informal activities, it must rely on an inflation tax to raise revenue and finance its expenditures. In our model, the government is able to imperfectly collect tax revenue from the informal sector, relying less on seigniorage revenue.

Policies that create incentives to work in the formal sector and improve enforcement of tax legislation decrease both the optimal interest rate and inflation. Either a higher probability of detection or more generous benefits affect optimal monetary policy by reducing distortions in the economy. Benefits, however, have a greater impact on the

\(^{10}\)See Perry et al. (2007).
optimal interest rate. In other words, the optimal interest rate is more elastic relative to benefits than to the detection probability. For instance, if we double the probability of detection relative to its baseline value ($\pi = 0.0093$), the predicted interest rate drops from 2.54 to 2.03 percent for the United States. On the other hand, for benefits twice the baseline value ($b = 0.08$), the reduction is greater, from 2.54 to 1.86 percent. In the presence of informal activities and tax evasion, the results suggest that policies that reward work in the formal sector are more effective.

4 Policy Implications

The model considers two distortions, namely a costly tax enforcement system and tax evasion. In this section, we analyze the main determinants of the optimal interest rate and inflation when the Friedman Rule is not optimal. In other words, we ask if different tax enforcement policies and enforcement spending can explain deviations from the Friedman rule. Tax enforcement spending plays an important role as it reflects the distortions associated with agents’ non-compliance with tax legislation, which in turn affects the optimal monetary policy. Moreover, the tax evasion penalty provides an alternative to an inflation tax in economies with tax evasion and informal activities.

The optimal monetary policy depends on the enforcement technology, as well as on tax enforcement policies. A positive interest rate and inflation are optimal when enforcement spending is positive and tax enforcement is imperfect. If the government can only tax informal workers and tax evaders with a given (small) probability, it is optimal to increase the inflation tax and deviate from the Friedman rule. The tax on money, a positive inflation tax, is a way for the government to reduce distortions from the informal sector and to compensate for tax revenue not collected from these activities. We quantify the effects of tax enforcement policies to the extent to which optimal monetary policy should deviate from the Friedman rule. In previous sections, we have shown that optimal inflation and interest rates differ across countries. These results are mainly driven by differences in tax enforcement policies, given that we assume the same enforcement spending parameter
for all countries.

This model casts doubt upon the desirability and sustainability of the recent convergence in interest rates and inflation in OECD countries. Moreover, this quantitative exercise identifies a concern raised by Sargent and Wallace (1981) regarding the need for coordination of monetary and fiscal policies. Eurozone countries have coordinated their monetary policies through the adoption of a single currency and a central bank, but they have not yet coordinated their fiscal policies. The evidence for the lack of fiscal policy coordination is both direct ($\tau^F_t, b_t, \lambda_t, g, \frac{deficit}{GDP}$ and $\frac{debt}{GDP}$) and indirect (as our results suggest different $\phi$). The model clearly indicates that if countries have different tax enforcement policies ($\tau^F_t, b_t, \lambda_t$) and enforcement spending ($\phi$), then the optimal interest rate and inflation will be different.

Previous studies have attempted to explain deviations from the Friedman Rule in the presence of tax evasion and informal activities. In Nicolini (1998), the quantitative effect of tax evasion on the optimal monetary policy is small, even in economies with large underground sectors. Cavalcanti and Villamil (2003) show that the optimal inflation rate ranges from 0 to 22 percent, depending on the size of the informal sector. Yesin (2004) explores the relevance of tax collection costs (the cost to collect formal taxes). Her model performs well only for a small group of countries and the optimal interest rate ranges from 5 to 43 percent. In Koreshkova (2006), the optimal policy takes into account the inefficiency of the informal sector. However, the optimal inflation rate delivered by her model is much higher than those observed in the data, e.g., an optimal inflation rate of 80 percent per annum for economies with an informal sector and 70 percent for the formal sector. My model predicts a low optimal inflation rate even for countries with large informal sectors. For instance, Italy has an informal sector equivalent to 27 percent of the GDP and the optimal inflation rate is below 10 percent.

All four studies share a similar feature, i.e., the government fails to detect and tax informal activities and tax evaders. The tax system is incomplete and the government is constrained to set the tax on informal income equal to zero. The key feature of our

\footnote{The estimated (formal) tax collection costs for the small group of countries used by Yesin (2004) are quite different and higher than those provided by OECD (2004) and used in this study.}
model is to recognize that the government has the additional tools needed to deal with
tax evasion and informal activities, namely, evasion penalties, benefits for agents that pay
income taxes and the probability detection, which can be used to reduce fiscal distortions.
We take into account tax enforcement spending and derive optimal policy. The Friedman
rule result still fails, as would be expected, but optimal deviations are smaller.

5 Conclusions

In this paper we show that, in the presence of tax evasion and informal activities, the
optimal monetary policy takes into account tax enforcement policies and spending on
enforcement. Positive inflation and interest rates are optimal when enforcement spending
is positive and tax enforcement is imperfect, i.e., the Friedman rule is not optimal. Tax
enforcement policies, namely, benefits, evasion penalties and collection spending, can
justify modest deviations from the Friedman Rule, i.e. evasion penalties and benefits can
reduce distortions.

In an economy with tax evasion, the main policymaking lesson drawn from our study
is that improved tax collection policies and procedures are relevant to monetary policy.
The enforcement spending parameter and tax enforcement policies play an important role
in determining optimal monetary policy and they vary across countries. This quantitative
exercise identifies a concern regarding the need for coordination of monetary and fiscal
policies. Eurozone countries have coordinated their monetary policies, but still lack
fiscal policy coordination. The model clearly indicates that if countries have different
tax enforcement policies and enforcement spending, then the optimal interest rate and
inflation will be different.
References


Appendix

Data and Calibrated Parameters

Table A.1 - Tax Enforcement Policies, Production Technology Parameter ($\alpha$) and Collection Technology Parameter ($\phi$) - OECD countries.

<table>
<thead>
<tr>
<th>Country</th>
<th>$l_t^{F}/l_t^I$</th>
<th>$g$</th>
<th>$\tau^F$</th>
<th>$b$</th>
<th>$\tau^F - b$</th>
<th>$\lambda_{MIN}$</th>
<th>$\lambda_{MAX}$</th>
<th>$s^F$</th>
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Notes: (1) Size of the informal sector (% GDP) $l_t^{F}/l_t^I$ (Schneider and Klinglmair, 2004); (2) Size of the government (%GDP); (3) "All-in" marginal tax rate for employees ($\tau^F$) includes personal income tax and employee social security contributions and less cash benefits for workers earning 100 per cent of the average wage level or Average Production Worker (APW), (OECD, 2004); (4) Cash transfers ($b_t$) include seven groups of benefits that are available to families: Unemployment benefits; Social assistance benefits; Cash housing benefits; Family benefits; Childcare benefits; Lone-parent benefits; Employment-conditional benefits (OECD, 2004a); (5) Administrative evasion penalty proportional to tax evade range: $\lambda_{MIN}-\lambda_{MAX}$ (OECD (2004)); (6) Formal tax collection cost $s^F$, OECD (2004). (n.a.) not available.
Table A.2 - Size of the Informal Sector - OECD countries.

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<th>Model</th>
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</table>

Sensitivity Analysis

Figure A.1 - Elasticity of Substitution Parameter $\rho$ and Optimal Interest Rate

Figure A.2 - Detection Probability $\pi$ and Optimal Interest Rate

Note: Baseline values: $\rho = 0.71$ and $\pi = 0.0093$. All other variables are at the U.S. level.
Figure A.3 - Benefits $b$ and Optimal Interest Rate

Note: Baseline value: $b = 0.08$. All other variables are at the U.S. level.

Figure A.4 - Risk-aversion term $v$ and Optimal Interest Rate