Optimal Monetary and Audit Policy
with Imperfect Taxation

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Abstract

We study optimal monetary and fiscal policy in the presence of informal activities and tax evasion in a cash-and-credit model where identical households are audited to determine their compliance with the tax code. Taxation of informal labor is imperfect, but the government has the tools needed to deal with informal activities and can choose them optimally to reduce fiscal distortions. We characterize both the optimal monetary (optimal interest rate) and fiscal policy (optimal income tax, evasion penalty and audit probability). When auditing is costless, a nominal interest rate equal to zero is optimal and attained when all agents are audited and both types of labor are taxed at the same rate. In the presence of auditing costs, the optimal audit policy does not implement the Friedman rule, and we report the welfare costs of implementing this monetary policy prescription. We derive conditions under which the Friedman rule can be recovered in an economy with informal activities.

Keywords: Monetary and Fiscal Policy; Friedman Rule; Informal Economy; Tax Evasion.

JEL Classification: E26; E52; E62; E63; H21; H26.

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1 Introduction

A central issue in monetary theory is the question of the optimal interest rate. Friedman (1969) argued that the optimal monetary policy is characterized by a nominal interest rate equal to zero. Many studies focus on the conditions under which this policy, known as the Friedman rule, is optimal. We study optimal monetary and fiscal policy in the presence of informal activities and tax evasion in a cash-and-credit model where identical households are audited to determine their compliance with the tax code. Taxation of informal labor is imperfect. When auditing is costless, the Friedman rule is the optimal monetary policy, which is attained when all agents are audited and informal and formal labor are taxed at the same rate, as efficiency requires. In the presence of auditing costs, the optimal audit rate and evasion penalty do not implement the Friedman rule, and the optimal monetary policy is to deviate from it. When informal activities are only imperfectly taxed and auditing is costly, we show that there are welfare costs of implementing the Friedman rule prescription. A key feature of our model is the recognition that the government has the tools needed to deal with informal activities and can choose them optimally to reduce fiscal distortions.

It has been shown that in the presence of an informal sector or tax evasion the Friedman rule is not optimal. Nicolini (1998) studies the optimal monetary policy in an economy with an underground sector where cash is used for transactions. Money is introduced by means of a cash-in-advance constraint, and the Friedman rule is not the optimal monetary policy. Introducing money via a shopping-time constraint, Cavalcanti and Villamil (2003) show that in the presence of informal activities, the Friedman rule is not optimal and the government raises revenue through a second-best inflationary tax. Koreshkova (2006) investigates quantitatively the importance of the public-finance motive for inflation in the presence of a tax-evading sector and finds a strong negative relationship between the size of the underground economy and the inflation rate.

Another frequently advocated reason to deviate from the Friedman rule is the presence of tax collection costs (see Aizenmann, 1983; Vegh, 1989). De Fiore (2000) explores conditions under

\footnote{There is a growing literature on the impact of the informal sector on macroeconomy and resource allocation. See for instance, Schneider and Enste (2000), Friedman et al. (2000), Fugazza and Jacques (2003), and Choi and Thum (2005).}
which the Friedman rule is optimal despite costs of collecting taxes. Yesin (2004) studies an economy with an informal sector and shows that costs of collecting formal income taxes can partly explain the observed deviations from the Friedman rule across countries.

Several other papers also investigate the optimality of the Friedman rule in overlapping generation models (Gahvari, 1998; Bhattacharya and Haslag, 2003; Bhattacharya et al., 2005; Palivos, 2005; Gahvari, 2007), in the context of a small open economy (Schmitt-Grohé and Uribe, 2003; Arsenneau, 2007; Cunha, 2008) and in the presence of heterogenous agents (Albanesi, 2002; da Costa and Werning, 2008). Correia and Teles (1996) and De Fiore and Teles (2003) discuss the relevance of the transactions technology in the determination of the optimal monetary policy. Chari et al. (1991, 1996), Correia and Teles (1999) and Ellison and Rankin (2007) study optimal monetary policy in different monetary models when the government must finance its expenditures without access to lump-sum taxation. Chari and Kehoe (2006) emphasize that the robust finding is not that nominal interest rates should be literally zero, but that nominal interest rates and inflation rates should be low.

The model is built on Lucas and Stokey (1983) and Chari, Christiano and Kehoe (1991, 1996). The economy is populated by a large number of ex-ante identical agents that can work in two sectors of the economy, the formal or informal one. Labor is the only factor of production in this economy. Output is produced using both formal and informal labor (or, output can be thought of as a composite of the output of the two sectors). Instruments to tax formal labor and punish tax evaders are available. The government uses proportional income taxes in the formal sector to finance its purchases. Agents try to evade taxes by working in the informal sector. Informal activities are defined as all income generating activities which do not comply with the tax obligations, i.e. tax evasion and non-compliance with economic legislation are assumed to be the main activities involved in it. We use the terms tax evasion and informal activities interchangeably. The government audits a certain fraction of the population and imposes an evasion penalty proportional to the tax evaded. We assume that agents are ex-ante identical and thus all provide labor services in the informal sector. Auditing everybody is not optimal because it is costly.

Chari and Kehoe (1999) defined an economy’s tax system as complete if the number of tax rates
the social planner can select is equal to the number of commodities in question, and incomplete if
the number of tax instruments is smaller than the number of commodities. Because not everyone
is audited and thus taxed on the informal labor services, we treat this tax system as incomplete.
The very nature of the informal sector makes it impossible for households to insure themselves
against the audit risk. This market incompleteness is a main driving force behind our results.

This paper departs from the existing literature in two key aspects. First, unlike most of the
previous work, the source of idiosyncratic shocks in our economy is the government and its imper-
fected taxation of informal labor income – in any period of time only a fraction of the population is
audited. That is, the government audit policy, which is equivalent to an incomplete tax system,
creates heterogeneity in the otherwise homogenous population. Second, we characterize both the
optimal monetary and fiscal policy, that is, the optimal interest rate and the optimal income tax,
evasion penalty and audit probability, respectively. This feature of our model enriches the discus-
sion about optimal policies when labor distortions exist. We extend the results for the optimality
of the Friedman rule obtained in the existing literature and, in particular, derive conditions under
which this monetary policy can be recovered in an economy with informal activities.

Our main findings are as follows. When the audit cost is zero and the audit policy (the audit
rate and the penalty rate) is fixed, the Friedman rule may or may not be the optimal monetary
policy. If the government can optimize the audit policy, the Friedman rule is recovered. The
Friedman rule is not optimal in this environment as long as the audit cost is positive. It is only
in the limiting case of zero audit cost that the Friedman rule becomes optimal. The intuition of
this limiting case is that when audit is costless it is optimal to audit everybody. Then the penalty
rate plays the role of the usual tax rate, and the tax system becomes complete and perfect. The
optimality of the Friedman rule is well known in this kind of environment (e.g., see Chari and
Kehoe, 1999). However, when the cost of audit is positive, the benefit of auditing more people
should be traded off against the cost of this activity. Since auditing everybody is not optimal, the
tax system becomes imperfect and incomplete. The audit introduces uncertainty, which, other
things being equal, lowers welfare. The government’s inability to perfectly tax informal labor

\footnote{Previous studies have assumed that the government cannot observe and tax transactions in the informal sector (see for instance, Nicolini, 1998; Cavalcanti and Villamil, 2003; Yesin, 2004; Koreshkova, 2006).}
leads to deviation from the Friedman rule. In other words, with the optimal fiscal and audit instruments, it is welfare enhancing to have a positive nominal interest rate. There is a welfare cost of attaining the zero level of nominal interest rate. When auditing is costly, we demonstrate that the optimal nominal interest rate is positive and, in order to recover the Friedman rule, either the audit rate or the penalty rate should be increased. We illustrate it quantitatively.

We observe an inverse relationship between the optimal audit rate and the optimal evasion penalty. As the cost of audit increases, the optimal audit rate falls and the optimal penalty for tax evasion rises. This result is consistent with an extensive literature on tax evasion and enforcement. For instance, Becker (1968) argues that a government should set the penalty for evasion high and the costly monitoring probability low to maximize the \textit{ex ante} utility of a representative agent. This policy deters evasion at minimal cost. The tax law literature also advocates this policy because it economizes on enforcement costs without sacrificing deterrence and raises revenue, according to Polinsky and Shavell (2005). The policymaker can save resources and achieve the same level of compliance by increasing penalties imposed only on dishonest taxpayers, and reducing the audit rate.

The optimal nominal interest rate is decreasing in the audit rate. When instruments to detect and tax informal labor are not available, it is optimal to deviate from the Friedman rule and, in this environment, inflation is an alternative and second-best tax. This intuition is similar to that found in Cavalcanti and Villamil (2003). As the audit rate increases, the government revenue collected from informal work earnings also increases, and it has to rely less on an inflationary tax, which reduces the optimal nominal interest rate. The rising cost of audit causes the optimal nominal interest rate, as well as the tax rate on formal labor, to rise. It leads to a shift of labor from the formal sector to the informal sector and a fall in welfare.

The paper is organized as follows: Section 2 presents the economy and the structure of the model and states the Ramsey problem. In Section 3, we characterize the optimal monetary and audit policy in the presence of tax evasion and audit cost and consider the optimality of the Friedman rule in this environment. Section 4 presents a numerical exercise, and Section 5 offers concluding comments.
2 A Cash-Credit Economy with Imperfect Taxation

We consider an infinite horizon monetary economy populated by a large number of identical agents. The economy consists of the formal and informal sectors. The tax authority randomly audits fraction $\theta$ of the population. In each period $t = 0, 1, 2, \ldots$, the agent can be either audited (state $a$), the probability of which is $\theta$, or not audited (state $n$), the probability of which is $1 - \theta$.

In period $t$, the agent is endowed with one unit of time which can be spent on formal work $l^F$, informal work $l^I$, or leisure $h$. Labor is the only factor of production in this economy. The final consumption good is a composite produced by both sectors, where total output is given by the constant returns to scale production function $F(l^F_t, l^I_t)$. We assume the Inada conditions on $F(l^F_t, l^I_t)$. Competitive pricing ensures that workers are paid their marginal products, $w^F_t = F_1(l^F_t, l^I_t)$ and $w^I_t = F_2(l^F_t, l^I_t)$, where $F_1(l^F_t, l^I_t)$ and $F_2(l^F_t, l^I_t)$ denote the marginal products of formal and informal labor, and $w^F_t$ and $w^I_t$ are the wage rates for formal and informal labor, respectively.

There are two consumption goods: a cash good $c_1$ and a credit good $c_2$. They are distinguished solely by the means of payment, which will be determined by a cash-in-advance constraint.

The timing of the model is as follows. At the beginning of each period, the household chooses $l^F_t$ and $l^I_t$ and works in the two sectors. Next, consumption $c_{1t}$ of the cash good is chosen based on a cash-in-advanced constraint. Consumption of the credit good is chosen based on the anticipated earnings and possible audit and consists of two parts. The first part, $c_{2t}^{\text{com}}$, is committed, whereas the second part, $c_{2t}^{\text{opt}}$, is optional and would be canceled if the agent is audited. To simplify the problem we assume that the value of the optional part $c_{2t}^{\text{opt}}$ is chosen to equal the penalty collection in the case of audit. Next, the household is paid for the labor services and receives interest earnings on bonds. The household purchases bonds $B_t$ for the next period. Then audit takes place, and the state $s \in \{a, n\}$ is determined. Payment for the credit good is made. Depending on the state $s$, the optional part of the credit good is either canceled (if $s = a$), or paid for and consumed (if $s = n$). The money holding $M_t$ for the next period is determined. The credit good consumptions are denoted $c_{2t}(a)$ and $c_{2t}(n)$ for cases $s = a$ and $s = n$ respectively.
The consumer’s problem is to maximize expected discounted lifetime utility

$$\max_{\{c_{1t}, c_{2t}, l_{t}, h_t, M_t, B_t\}} \sum_{t=0}^{\infty} \beta^t \left[ \theta U(c_{1t}, c_{2t}(a), h_t) + (1 - \theta) U(c_{1t}, c_{2t}(n), h_t) \right]$$

subject to

1. $$p_t[c_{1t} + c_{2t}(a)] + M_{t+1} + B_{t+1} \leq R_t B_t + M_t + p_t[\tau_F^t w_t F^F + (1 - \rho_t) w_t l_{t}^F] + A_t$$
2. $$p_t[c_{1t} + c_{2t}(n)] + M_{t+1} + B_{t+1} \leq R_t B_t + M_t + p_t[\tau_F^t w_t F^F + w_t^I] + A_t$$
3. $$p_t c_{1t} \leq M_t$$
4. $$l_{t}^F + l_{t}^I + h_t = 1$$
5. $$-B \leq B_t/p_t \leq B$$
6. $$M_t, c_{1t}, c_{2t}(a), c_{2t}(a), h_t, l_{t}^I, l_{t}^F \geq 0$$

where $$h_t$$ is leisure, $$R_t$$ the interest rate paid on bonds, $$p_t$$ the price level in the economy, $$\tau_F^t$$ the tax rate on formal labor income, and $$\rho_t$$ the tax evasion penalty rate. Bond holdings $$B_t$$ are bounded by some fixed number $$B$$ to avoid Ponzi schemes. The penalty for tax evasion is bounded from above, that is, $$\rho_t \in (0, \rho_M)$$, where $$\rho_M$$ is the maximal penalty level that the government can impose on a tax evader. The discount factor $$\beta \in (0, 1)$$, and $$U$$ is a strictly concave, twice continuously differentiable function that is separable in consumption and leisure and satisfies the Inada conditions.

The government finances an exogenously given stream of expenditures $$\{g_t\}$$ through printing new money, issuing new bonds, collecting labor income taxes from formal workers, enforcing the tax code and imposing penalties on informal workers. The government’s period budget constraint is

$$p_t g_t + R_t B_t = B_{t+1} + M_{t+1} - M_t + p_t \tau_F^t w_t F^F + p_t \theta \rho_t w_t^I - \theta K,$$

where $$K$$ is a fixed unit cost of audit. The left-hand side of this equation contains government expenditures $$g_t$$ and current period debt service. The terms on the right side are, respectively, government revenues generated by asset sales, formal tax revenue and informal tax revenue.
The resource constraint in this economy is

\[ c_{1t} + \left[ \theta c_{2t}(a) + (1 - \theta)c_{2t}(n) \right] + g_t = F(l^F_t, l^I_t) - \theta K. \] (4)

Let \( x_t = (c_{1t}, c_{2t}(a), c_{2t}(n), l^F_t, l^I_t, M_t, B_t) \) denote period \( t \) allocation, and \( x = (x_t) \). We let \( q = (p_t, R_t) \) denote a price system for this economy and \( \pi = (\tau^F_t, \rho_t, \theta) \) denote government policy; here \( \tau^F_t \) represents fiscal policy and \( \theta, \rho_t \) the government’s audit policy. The initial stock of money \( M_{-1} \) and the initial stock of nominal debt \( B_{-1} \) are given. Given this description of the economy, we now define competitive equilibrium.

**Definition 1** A competitive equilibrium is a policy \( \pi \), an allocation \( x \), and a price system \( q \) such that given the policy and the price system, the resulting allocation maximizes the consumer’s utility and satisfies the government’s budget constraint, the economy’s resource constraint, and market clearing conditions.

Let \( \beta^t \theta \lambda_{at}, \beta^t (1 - \theta) \lambda_{nt} \) and \( \beta^t \nu_t \) be the Lagrange multipliers for consumer budget constraints (1), (2), and CIA constraint (3), respectively. Let \( U(s, t) = U(c_{1t}, c_{2t}(s), h_t) \), and likewise for \( U_i(s, t) \), where \( s = a, n \). The first-order conditions with respect to \( c_{1t}, c_{2t}(s), l^F_t, l^I_t, M_t, \) and \( B_t \) are as follows:

\[
\begin{align*}
[\theta U_1(a, t) + (1 - \theta)U_1(n, t)] & - [\theta \lambda_{at} + (1 - \theta)\lambda_{nt} + \nu(s^t)] p_t = 0, \\
U_2(s, t) - \lambda_{at} p_t &= 0, \\
- [\theta U_3(a, t) + (1 - \theta)U_3(n, t)] + p_t (1 - \tau^F_t) w^F_t [\theta \lambda_{at} + (1 - \theta)\lambda_{nt}] &= 0, \\
- [\theta U_3(a, t) + (1 - \theta)U_3(n, t)] + p_t w^I_t [(1 - \rho_t)\theta \lambda_{at} + (1 - \theta)\lambda_{nt}] &= 0, \\
- [\theta \lambda_{at} + (1 - \theta)\lambda_{nt}] + \beta [\theta \lambda_{a,t+1} + (1 - \theta)\lambda_{n,t+1}] + \beta \nu_{t+1} &= 0, \\
- [\theta \lambda_{at} + (1 - \theta)\lambda_{nt}] + \beta [\theta \lambda_{a,t+1} + (1 - \theta)\lambda_{n,t+1}] R_{t+1} &= 0.
\end{align*}
\]
The equilibrium conditions can be represented as

\[ 1 - \tau_t^F = \frac{E_t(U_3)}{E_t(U_2)} \cdot \frac{1}{F_1(l_t^b, l_t^I)}, \]

\[ \rho_t = \frac{E_t(U_2)F_2(l_t^F, l_t^I) - E_t(U_3)}{\theta U_2(a, t)F_2(l_t^F, l_t^I)}, \]

\[ R_{t+1} = \frac{E_{t+1}(U_1)}{E_{t+1}(U_2)}, \]

where \( E_t(U_i) = \theta U_i(a, t) + (1 - \theta)U_i(n, t) \). Condition (6) can be also written as

\[ E_t(U_3) - F_2(l_t^F, l_t^I)[E_t(U_2) - \theta \rho_t U_2(a, t)] = 0. \]

Money earns a gross nominal return of 1. Since \( R_{t+1} \geq 1 \), in any equilibrium, the following constraint must hold:

\[ E_{t+1}(U_1) \geq E_{t+1}(U_2). \]

At the beginning of each period, the government announces its program of tax rate, evasion penalty and expenditures, and individuals behave competitively. The objective of the social planner is to choose values of the fiscal instruments such that the agent’s utility is maximized. The problem is constrained by the private sector’s optimization behavior and by the budget of the government. The social planner does not directly control the agent’s allocations, and the problem is of second-best because the social planner chooses the fiscal instruments that satisfies the optimization restrictions of the private agent, i.e. the first-order conditions of the private agent’s problem.

The Ramsey problem is a programming problem of finding optimum within a set of allocations that can be implemented as a competitive equilibrium with distorting taxes; the latter set is characterized by a resource constraint and an implementability constraint (see Chari and Kehoe, 1999). To derive the implementability constraint, we multiply the period \( t \) budget constraints (1) and (2) by their Lagrange multipliers \( \beta^t \theta \lambda_{at} \) and \( \beta^t(1 - \theta) \lambda_{nt} \) respectively, multiply the period \( t \) cash-in-advance constraint by its Lagrange multiplier \( \beta^t \nu_t \) for each period \( t \), and sum over \( t \). All the money terms except \( M_{-1} \) and all the bond terms except \( B_{-1} \) will cancel out because of the
use of the first-order conditions of the agent’s optimization problem. This yields the following:

\[
\sum_{t=0}^{\infty} \beta^t \left\{ \theta \left[ U_1(a, t)c_{1t} + U_2(a, t)c_{2t}(a) - U_{3t}(1 - h_t) \right] \\
+ (1 - \theta) \left[ U_1(n, t)c_{1t} + U_2(n, t)c_{2t}(n) - U_{3t}(1 - h_t) \right] \right\} = 0. \tag{9}
\]

As has been mentioned earlier, we study an economy where the government has the fiscal instrument to tax informal activities, but in any period of time only a fraction of the population is audited and thus taxed. Hence, the competitive equilibrium condition (7) has to be imposed explicitly in the Ramsey Problem. Equation (8) is a feature of the competitive equilibrium that also constrains the set of Ramsey allocations.

**Proposition 1 (Ramsey Allocations)** The consumption and labor allocations in a competitive equilibrium satisfy (4), (7), (8) and the implementability constraint (9). Furthermore, allocations that satisfy (4), (7), (8) and (9) can be decentralized as a competitive equilibrium.

**Proof.** Suppose allocation \( x = (c_{1t}, c_{2t}(a), c_{2t}(n), l^F_t, l^I_t) \) is a competitive equilibrium allocation. It is obvious that it satisfies (4), (7), (9), and (8). Suppose allocation \( x = (c_{1t}, c_{2t}(a), c_{2t}(n), l^F_t, l^I_t) \) satisfies (4), (7), (8) and (9). We have to show that we can find prices \( (p_t, w^F_t, w^I_t, R_t) \), multipliers \( (\lambda_{at}, \lambda_{nt}, \nu_t) \), and money and bond vectors \( (M_t, B_t) \) such that they satisfy the first-order conditions of the household problem, budget constraints and CIA constraints of households.

Choose \( w^F_t = F_1(l^F_t, l^I_t) \), and \( w^I_t = F_2(l^F_t, l^I_t) \). Since the F.O.C.s for consumptions, money and bonds imply

\[
R_{t+1} = \frac{E_{t+1}(U_1)}{E_{t+1}(U_2)},
\]

choose \( R_t \) as in this formula above. The F.O.C.s for credit goods and bonds imply that

\[
R_{t+1} = \frac{1}{\beta} \cdot \frac{p_t}{p_{t+1}} \cdot \frac{E_t(U_2)}{E_{t+1}(U_2)}.
\]

Thus, choose the price ratio \( p_{t+1}/p_t \) as

\[
\frac{p_{t+1}}{p_t} = \frac{1}{\beta R_{t+1}} \cdot \frac{E_t(U_2)}{E_{t+1}(U_2)}.
\]
Choose $M_0 = 1$, and then choose the value of $p_0$ to satisfy the CIA constraint $p_0 c_{10} = M_0$. Then the value of $p_0$ and the sequence of price ratios $p_{t+1}/p_t$ would determine the entire price sequence. Choose $\lambda_{st}$ to satisfy the F.O.C. for the credit goods. Given these values, choose $\nu_t$ to satisfy the F.O.C. for the cash good. Choose the value of $\tau^F_t$ to satisfy the F.O.C. for $l^F_t$, and that of $\rho_t$ to satisfy the F.O.C. for $l^I_t$. Choose $M_t$ for $t \geq 1$ to satisfy the CIA constraints. Finally, choose $B_t$ to satisfy the household budget constraint. The chosen allocations and prices satisfy all the conditions of competitive equilibrium.

We will assume constant government spending over time, i.e. $g_t = g$, $t = 0, 1, \ldots$, and write the steady state Ramsey problem as follows.

$$\max_{c_1, c_2(a), c_2(n), F^L, l^I} \left[ \theta U(a) + (1 - \theta) U(n) \right]$$

subject to

$$\{ \theta [U_1(a)c_1 + U_2(a)c_2(a) - U_3(a)(l^F + l^I)]$$

$$+ (1 - \theta) [U_1(n)c_1 + U_2(n)c_2(n) - U_3(n)(l^F + l^I)] \} = 0,$$

(10)

$$[\theta U_3(a) + (1 - \theta) U_3(n)] - F_2(l^F, l^I) [\theta U_2(a) + (1 - \theta) U_2(n) - \theta \rho U_2(a)] = 0,$$

(11)

$$F(l^F, l^I) - [c_1 + \theta c_2(a) + (1 - \theta)c_2(n)] - g - \theta K = 0,$$

(12)

$$[\theta U_1(a) + (1 - \theta) U_1(n)] - [\theta U_2(a) + (1 - \theta) U_2(n)] \geq 0,$$

(13)

$$c_2(n) - c_2(a) - \rho F_2(l^F, l^I) l^I = 0.$$

(14)

Here (10) is the implementability constraint corresponding to (9), (11) is an analogue of condition (7), (12) is the resource constraint and corresponds to (4), (13) corresponds to (8), and (14) states that the difference in credit good consumption between an audited household and a household that is not audited equals the amount of penalty paid to the government by the audited household. We will omit the constraint $E(U_1) \geq E(U_2)$ in this problem and determine whether this constraint would bind if imposed. Let $\psi, \gamma, \eta,$ and $\zeta$ be the Lagrange multipliers on the implementability constraint (10), condition (11), resource constraint (12), and constraint (14), respectively.
first-order conditions for \( c_1, c_2(s), l^F \) and \( l^I \) are, respectively:

\[
E(U_1) + \psi E[U_{11} c_1 + U_1 + U_{21} c_2 - U_{31}(l^F + l^I)] \\
+ \gamma \{ E(U_{31}) - F_2[E(U_{21}) - \theta \rho U_{21}(a)] \} - \eta = 0, \tag{15}
\]

\[
U_2(a) + \psi [U_{12}(a)c_1 + U_2(a) + U_{22}(a)c_2(a) - U_{32}(a)(l^F + l^I)] \\
+ \gamma [U_{32}(a) - F_2 U_{22}(a)(1 - \rho)] - \eta - \zeta / \theta = 0, \tag{16}
\]

\[
U_2(n) + \psi [U_{12}(n)c_1 + U_2(n) + U_{22}(n)c_2(n) - U_{32}(n)(l^F + l^I)] \\
+ \gamma [U_{32}(n) - F_2 U_{22}(n)] - \eta + \zeta / (1 - \theta) = 0, \tag{17}
\]

\[
- E(U_3) + \psi E[-U_{13} c_1 - U_{23} c_2 + U_{33}(l^F + l^I) - U_3] + \eta F_1 - \zeta \rho F_{21} l^I \\
+ \gamma \{ - E(U_{33}) - F_{21}[E(U_2) - \theta \rho U_2(a)] + F_{22}[E(U_{23}) - \theta \rho U_{23}(a)] \} = 0,
\]

3 Optimal Monetary and Audit Policy

In this section, we study the optimal monetary and audit policies in the presence of tax evasion and incomplete taxation of informal labor income. In this economy, the Friedman rule is not necessarily the optimal monetary policy. Moreover, the fraction of the population that is audited in any period of time plays an important role in determining deviations from the Friedman rule result. The government’s consumption expenditures, \( g \), are assumed to be constant, and we study this economy in a steady state.

As in Chari et al. (1996), we consider utility functions of the form:

\[
U(c_1, c_2, h) = W(w(c_1, c_2), h),
\]

where \( w \) is homothetic. Functions of this form satisfy the following conditions:

\[
\frac{E(U_{11}.c_1)}{E(U_1)} = \frac{E(U_{12}.c_1)}{E(U_2)}, \quad \frac{U_{31}}{U_1} = \frac{U_{32}}{U_1} = \frac{W_{12}}{W_1}.
\]
We make the following assumption about the utility function $U$:

$$\frac{U_1}{U_2} > \frac{U_{21}}{U_{22}}.$$  \hfill (18)

Note that if $U$ is additively separable in $c_1$ and $c_2$, then $U_{21} = 0$, and (18) holds. Also, since $U_2 > 0$ and $U_{22} < 0$, the above condition can be rewritten as $U_1 U_{22} < U_2 U_{21}$.

The source of idiosyncratic shocks in this economy is the government’s imperfect taxation of informal labor income. The government audit policy determines the optimal monetary policy and its deviations (if any) from the Friedman rule. That is, the optimal monetary policy in an economy where taxation is incomplete depends on the fraction of the population that is audited by the government (i.e., the optimal interest rate $R$ depends on $\theta$). Moreover, the larger the fraction of the population audited by the government, the smaller the deviation from the Friedman result. The incomplete taxation feature of our economy is captured by equation (7), and its multiplier $\gamma$ plays an important role in the determination of the optimal monetary policy in this economy.

The optimality condition (7) from the household problem implies that the informal labor choice equates the marginal utility of consumption gained as a result of more work in the informal sector (the audit adjusted marginal utility: $F_2[\theta U_2(a) + (1-\theta)U_2(n) - \theta pU_2(a)]$) to the marginal utility of leisure $U_3$ given up as a result of more work in the informal sector. Notice that the marginal utility of consumption derived from informal work is decreasing in $\theta$. If we increase the right hand side of constraint (7) from zero to a small positive number, i.e. if we drive a positive wedge between the marginal utility of leisure and the marginal utility of informal work, the resulting value of utility changes. This value of utility increases if $\gamma$ is positive, and decreases otherwise.

If $\theta = 0$, the formal sector is taxed, and the informal one is not. The distortionary tax in the formal sector drives the after-tax wage $(1 - \tau^F)w^F$ lower than the efficient level. In equilibrium, the wage rate in the informal sector equals the after-tax wage level in the formal sector. Since the informal sector is not distorted, from efficiency point of view it would be desirable to increase $l^I$. By slightly increasing the right hand side of (7), the social planner can increase welfare. Thus, the associated multiplier $\gamma$ is positive, which renders $R > 1$. In this case, the government has no instruments to tax one of the commodities in the economy, and as a consequence, the Friedman
rule is not the optimal monetary policy (Nicolini, 1998; Cavalcanti and Villamil, 2003).

When \( \theta = 1 \) and \( \rho > \tau^F \), the informal sector is taxed more heavily than the formal one. Efficiency would require relatively less work in the informal sector and more in the formal one. Thus, \( \gamma < 0 \), which implies \( R = 1 \). In this case when the government perfectly tax informal activities, our model is compatible with the ones in Chari et al. (1996) and Jones, Manuelli and Rossi (1997), where formal and informal labor can be viewed as two different labor inputs (in an economy with no informal sector).

Thus, when the penalty rate \( \rho \) is fixed and audit rate \( \theta \) varies between 0 and 1, the multiplier \( \gamma \) changes its sign from positive to negative. It is easy to see using similar argument that once \( \gamma \) turns negative, it never becomes positive.

**Proposition 2** Suppose that evasion penalty \( \rho \) is fixed. If the utility function is separable between consumption and leisure, and only an exogenously given fraction \( \theta \in (0,1) \) of the population is audited, then the Friedman rule is not necessarily the optimal monetary policy.

**Proof.** Let us split the F.O.C. (15) into two pieces as follows. It can be written as \( A + (-A) = 0 \), where \( A \) is the part corresponding to “audit” and \( (-A) \) the part corresponding to “no audit”. Then

\[
U_1(a) + \psi[U_{11}(a)c_1 + U_1(a) + U_{21}(a)c_2(a) - U_{31}(a)(I^F + I^I)]
+ \gamma[U_{31}(a) - F_2U_{21}(a)(1 - \rho)] - \eta = \frac{A}{\theta};
\]

\[
U_1(n) + \psi[U_{11}(n)c_1 + U_1(n) + U_{21}(n)c_2(n) - U_{31}(n)(I^F + I^I)]
+ \gamma[U_{31}(n) - F_2U_{21}(n)] - \eta = -\frac{A}{1 - \theta}.
\]

Here the value of \( A \) is not important since in our argument below it will vanish. If we multiply the two equations above by \( \theta \) and \( 1 - \theta \) respectively, we obtain (15). Divide these two conditions
by $U_1(a)$ and $U_1(n)$, respectively:

$$(1 + \psi) + \psi \left[ \frac{U_{11}(a)c_1 + U_{21}(a)c_2(a)}{U_1(a)} - \frac{U_{31}(a)}{U_1(a)}(l^F + l^I) \right] + \gamma \frac{U_{31}(a)}{U_1(a)} = \eta + \frac{\gamma F_2 U_{21}(a)(1 - \rho)}{U_1(a)} + \frac{A}{\theta(U_1(a))};$$

(19)

$$\begin{align*}
(1 + \psi) + \psi \left[ \frac{U_{11}(n)c_1 + U_{21}(n)c_2(n)}{U_1(n)} - \frac{U_{31}(n)}{U_1(n)}(l^F + l^I) \right] + \gamma \frac{U_{31}(n)}{U_1(n)} &= \eta + \frac{\gamma F_2 U_{21}(n)(1 - \rho)}{U_1(n)} - \frac{A}{(1 - \theta)U_1(n)}.
\end{align*}$$

(20)

Similarly, divide the first-order conditions for $c_2(n)$ in equations (16) and (17) by $U_2(a)$ and $U_2(n)$ respectively:

$$\begin{align*}
(1 + \psi) + \psi \left[ \frac{U_{12}(a)c_1 + U_{22}(a)c_2(a)}{U_2(a)} - \frac{U_{32}(a)}{U_2(a)}(l^F + l^I) \right] + \gamma \frac{U_{32}(a)}{U_1(a)} &= \eta + \frac{\gamma F_2 U_{22}(a)(1 - \rho)}{U_2(a)} + \frac{\zeta}{\theta U_2(a)}; \\
(1 + \psi) + \psi \left[ \frac{U_{12}(n)c_1 + U_{22}(n)c_2(n)}{U_2(n)} - \frac{U_{32}(n)}{U_2(n)}(l^F + l^I) \right] + \gamma \frac{U_{32}(n)}{U_1(n)} &= \eta + \frac{\gamma F_2 U_{22}(n)(1 - \rho)}{U_2(n)} - \frac{\zeta}{(1 - \theta)U_2(n)}.
\end{align*}$$

(21)

(22)

The left-hand sides of equations (19) and (21) have the same value, which implies

$$U_2(a) \left[ \eta + \gamma F_2 U_{21}(a)(1 - \rho) + A/\theta \right] = U_1(a) \left[ \eta + \gamma F_2 U_{22}(a)(1 - \rho) + \zeta/\theta \right].$$

(23)

Similarly, from equations (20) and (22) we have

$$U_2(n) \left[ \eta + \gamma F_2 U_{21}(n) - A/(1 - \theta) \right] = U_1(n) \left[ \eta + \gamma F_2 U_{22}(n) - \zeta/(1 - \theta) \right].$$

(24)

Multiplying (23) and (24) by $\theta$ and $(1 - \theta)$, respectively, and adding them up we obtain:

$$\begin{align*}
\eta E(U_2) + \gamma F_2 E(U_1 U_2) - \theta \rho \gamma F_2 U_{21}(a) U_2(a) + A \left[ U_2(a) - U_2(n) \right] &= \eta E(U_1) + \gamma F_2 E(U_2 U_1) - \theta \rho \gamma F_2 U_{22}(a) U_1(a) + \zeta \left[ U_1(a) - U_1(n) \right].
\end{align*}$$
Dividing both sides by $\eta E(U_2)$ and rearranging we have

$$
\frac{E(U_1)}{E(U_2)} = 1 + \frac{\gamma F_2}{\eta E(U_2)} \left\{ E(U_{21}U_2 - U_{22}U_1) - \theta \rho [U_{21}(a)U_2(a) - U_{22}(a)U_1(a)] \right\}
+ \frac{A}{\eta E(U_2)} \left[ U_2(a) - U_2(n) \right] - \frac{\zeta}{\eta E(U_2)} \left[ U_1(a) - U_1(n) \right].
$$

Equation (25) is an expression for the interest rate $R$. Observe that $A = 0$ and $\zeta = 0$ when $\theta = 0$. This can be seen by multiplying (19) and (21) by $\theta$ and imposing $\theta = 0$. By similar arguments applied to (20) and (22) we conclude that $A = 0$ and $\zeta = 0$ when $\theta = 1$. Thus, in two extreme cases, when $\theta = 0$ and $\theta = 1$, the last two terms in (25) vanish. By assumption (18),

$$
E(U_{21}U_2 - U_{22}U_1) - \theta \rho [U_{21}(a)U_2(a) - U_{22}(a)U_1(a)] > 0.
$$

As discussed before, when $\theta = 0$, the multiplier $\gamma > 0$, and the right-hand side of equation (25) is greater than 1. In this case the Friedman rule is not optimal. On the other hand, when $\theta = 1$ and $\rho > \tau^F$, the right-hand side of (25) is less than one. Recall that we did not impose condition $E(U_1) \geq E(U_2)$ in the Ramsey problem. Clearly, this condition would bind if imposed. Thus $R = 1$, and the Friedman rule holds. By continuity, depending on the value of $\theta \in (0,1)$, the nominal interest rate $R$ can take on values both greater than one and equal to one. Thus, the Friedman rule is not necessarily optimal. 

The intuition for this result is as follows. In this economy, on one hand the government can tax formal income perfectly, but on the other hand taxation of informal activities is incomplete, and only a fraction of the population is audited. A standard result in the public finance literature is that if the utility function is separable in leisure and the subutility function over consumption goods is homothetic, then the optimal policy is to tax all consumption goods at the same rate. In our economy, the taxes on labor incomes (formal and informal) do not apply to consumption of the cash good and the credit good at the same rate. If $R = 1$, the cash good is taxed at a lower rate than the credit good. With such preferences, efficiency requires that $R > 1$ and, therefore, that monetary policy should deviate from the Friedman rule. For optimal $R > 1$, the cash good
is effectively taxed at the same rate as the credit good. Here, it is optimal to deviate from the Friedman rule, and in this environment inflation is an alternative and second-best tax. That is, the optimal policy is to set a positive inflation tax, in addition to two other distortionary tax instruments, namely a positive formal income tax ($\tau^F > 0$) and a positive evasion penalty ($\rho > 0$).

In many economic models in which money plays a role, welfare is maximized when the inflation rate is low enough so that the nominal interest rate is zero. At positive nominal interest rates, individuals incur a positive opportunity cost by holding money instead of bonds. Since the social cost of producing money is nearly zero, there is a divergence between the private and social costs of holding money when nominal interest rates are positive. Individuals choose to equate the marginal benefit of holding money with the private cost, so positive nominal interest rates generate inefficiency. If the government sets the nominal interest rate at zero, equating private and social costs, it can eliminate this inefficiency. In models where there are no labor market distortions (for instance, no informal activities), it follows that this same monetary policy is optimal from a welfare perspective.

In our economy, however, the optimal interest rate and optimal rate of inflation could be higher than that corresponding to the Friedman rule. In the analysis to this point the audit policy instruments (penalty for tax evasion $\rho$ and audit probability $\theta$) have been treated as given. But, if such instruments are available, the government can choose them optimally to maximize agents’ welfare. In short, the government can optimally deviate from the Friedman rule prescription and still maximize welfare, provided that audit and fiscal policies are chosen optimally.

Now we characterize the optimal audit policy instruments. Let $(c_1(\theta, \rho), c_2(a; \theta, \rho), c_2(n; \theta, \rho), l^F(\theta, \rho), l^I(\theta, \rho))$ denote the solution of the Ramsey problem for a fixed audit policy $(\theta, \rho)$, and $R(\theta, \rho), \tau^F(\theta, \rho), U(\theta, \rho)$ denote the corresponding values of the gross nominal interest rate, formal sector tax rate, and utility function. Consider the following problem of optimal audit policy:

$$\max_{\theta, \rho} E[U(c_1(\theta, \rho), c_2(\theta, \rho), l^F(\theta, \rho), l^I(\theta, \rho))].$$

Let $(\theta^*, \rho^*)$ and $U^* = U(\theta^*, \rho^*)$ be the solution of this problem and the corresponding value of the

---

3 In a different environment, Cavalcanti and Villamil (2003) provide a similar intuition for a positive inflation tax applied to the informal sector.
utility function respectively. We are interested in the solution of the Ramsey problem when the audit policy is chosen optimally. Let the solution of this problem be also starred in our notation: \( c_1^*, c_2^*(a), c_2^*(n), l^F*, l^I* \). The corresponding tax rate and interest rate are denoted \( \tau^F* \) and \( R^* \), respectively.

Let us introduce functions \( \theta(\rho) \) and \( \rho(\theta) \) as follows. For any fixed \( \theta \in [0, 1] \), define \( \rho(\theta) \) to be the best level of penalty, i.e.

\[
\max_{\rho} U(\theta, \rho) = U(\theta, \rho(\theta)).
\]

And, similarly, for any fixed \( \rho \in (0, \rho_M] \), where \( \rho_M \) is the maximal penalty level that the government can impose on a tax evader, define \( \theta(\rho) \) to be the best level of audit rate, i.e.

\[
\max_{\theta} U(\theta, \rho) = U(\theta(\rho), \rho).
\]

These functions satisfy \( \rho(\theta^*) = \rho^* \) and \( \theta(\rho^*) = \theta^* \).

If the government has no instruments to audit informal activities, \( \theta = 0 \), and \( \rho \) is no longer relevant. However, on the other extreme, when the government can perfectly observe and tax informal labor (\( \theta = 1 \)), there are two types of labor that are perfectly taxable and efficiency requires that the tax rates be equal: \( \rho = \tau^F \). Thus, \( \rho(1) = \tau^F(1, \rho(1)) \). The following proposition is an application of the uniform commodity taxation and it demonstrates the optimality of the Friedman rule in one special case when the government chooses its audit policy optimally.

**Proposition 3** Suppose the cost of auditing is zero. Then the welfare maximizing \((\theta^*, \rho^*)\) is such that the Friedman rule is the optimal monetary policy: \( R^* = 1 \).

**Proof.** When the government faces zero cost of auditing the economy, the optimal policy is to monitor the whole economy, i.e. \( \theta^* = 1 \). Recall that condition (5) for \( \tau^F \) has been substituted out before the Ramsey problem was formulated. Therefore, this condition is satisfied for every allocation that is a solution to the Ramsey problem. When \( \rho = \tau^F \), condition (6) and condition (5) coincide. Thus, imposing it in the Ramsey problem is redundant, and its multiplier \( \gamma = 0 \). That is, \( \rho(1) = \tau^F(1, \rho(1)) \), as we already argued informally before. Since the last two terms on the right-hand-side of (25) vanish, and \( \gamma = 0 \), we have \( R = 1 \).
Intuitively, optimality requires that the tax (penalty) on informal work be equal to the tax on formal work, $\rho^* = \tau^F$. The optimal audit rate $\theta^*$ eliminates imperfection of taxation, i.e. everybody is audited. The optimal evasion penalty $\rho^*$ minimizes labor market distortions. Together they render optimality of the Friedman rule: $R(\theta^*, \rho^*) = 1$.

The following proposition demonstrates the optimality of the Friedman rule when the audit policy is not optimal (the penalty rate $\rho$ is too high).

**Proposition 4** Suppose the cost of auditing is zero. Suppose all the population is audited, i.e. $\theta = 1$. If $\rho \in (\rho(1), \rho_M]$, then the Friedman rule is the optimal monetary policy.

**Proof.** Recall that the multiplier $\gamma = 0$ when $\rho = \rho(1)$ (see the proof of Proposition 3). If $\rho > \rho(1)$, then $\gamma < 0$: since the informal sector is taxed more heavily than the formal sector, it is desirable to decrease $l^I$ and increase $l^F$. Since the last two terms on right-hand side of (25) vanish, and $\gamma = 0$, we have $R = 1$. \qed

It is clear that if the government could perfectly tax informal activities ($\theta = 1$), the Friedman rule is the optimal monetary policy. We argue, however, that the Friedman rule can also be recovered even if the government does not audit the whole economy, i.e., for $\theta \in (0, 1)$. As deviations from the Friedman rule can be seen as a second-best tax on the economy, the imperfection of taxation of informal labor can be reduced if the government increases the fraction of the population that is audited. For a fixed $\rho \in (\tau^F, \rho_M)$, denote

$$\hat{\theta}(\rho) = \min\{\theta \in (0, 1) \mid R(\theta, \rho) = 1\}.$$ 

That is, $\hat{\theta}(\rho)$ is the smallest $\theta \in (0, 1]$ such that the Friedman rule holds. We will demonstrate that once the interest rate $\theta$ hits the lowest level 1, it stays there. In other words, for $\theta \in [\hat{\theta}(\rho), 1]$, $R(\theta, \rho) = 1$. As $\theta$ increases from zero to $\hat{\theta}$, the government revenue collected from informal work earnings increases and it has to rely less on an inflationary tax, until the point where $R(\hat{\theta}, \rho(\hat{\theta})) = 1$.

The next proposition formalizes this result by showing existence of $\hat{\theta}$.

**Proposition 5** Suppose the audit cost $K = 0$, and the evasion penalty $\rho \in (\tau^F, \rho_M)$ is fixed.
Then there exists such \( \hat{\theta}(\rho) \in (0, 1) \) that

\[
R(\theta, \rho) > 1 \quad \text{for} \quad 0 \leq \theta < \hat{\theta}(\rho), \\
R(\theta, \rho) = 1 \quad \text{for} \quad \hat{\theta}(\rho) \leq \theta \leq 1.
\]

**Proof.** From the analysis above it follows that \( \gamma > 0 \) when \( \theta = 0 \), and that \( \gamma < 0 \) when \( \theta = 1 \). Also, in these two extreme cases, the last two terms of equation (25) vanish. Let the right-hand side of equation (25) be denoted as \( f(\theta) \), that is:

\[
f(\theta) = 1 + \frac{\gamma F_2}{\eta E(U_2)} \left\{ E(U_{21}U_2 - U_{22}U_1) - \theta \rho [U_{21}(a)U_2(a) - U_{22}(a)U_1(a)] \right\} \\
+ \frac{\zeta}{\eta E(U_2)} \left\{ U_2(a) - U_2(n) - [U_1(a) - U_1(n)] \right\}
\]

Clearly, \( f \) is a continuous function of \( \theta \) and \( f(0) > 1 \) and \( f(1) < 1 \). Therefore, there exists a \( \hat{\theta} \in (0, 1) \) such that \( f(\hat{\theta}) = 1 \). Since, \( R = \max(f, 1) \), for \( \theta = \hat{\theta} \), the gross nominal interest rate \( R \) is equal to 1, i.e. the Friedman rule holds. As we argued earlier, the multiplier \( \gamma \), once it becomes negative, never turns positive. Thus, indeed

\[
R(\theta, \rho) > 1 \quad \text{for} \quad 0 \leq \theta < \hat{\theta}, \\
R(\theta, \rho) = 1 \quad \text{for} \quad \hat{\theta} \leq \theta \leq 1.
\]

When the cost of auditing the economy is positive (\( K > 0 \)), the optimal audit rate (\( \theta^* \)) and evasion penalty (\( \rho^* \)) that maximizes social welfare do not implement the Friedman rule (\( R^* > 1 \)). As in the case of \( K = 0 \), we can find optimal audit policy instruments (\( \theta^*, \rho^* \)) that maximize agents’ welfare, according to (26). It follows that if the government has to spend resources to monitor the economy, the optimal audit rate is less than one, i.e. \( \theta^* < 1 \). In this environment where the government has to give up some of the informal labor tax revenue, it is optimal for the government to set a positive nominal interest rate, deviating from the Friedman rule prescription. By doing so the government indirectly recovers part of its tax revenue losses in the informal sector. If the government instead wants to implement a monetary policy of zero nominal interest rate,
there will be a welfare loss. We illustrate it quantitatively.

4 Numerical Exercise

In this section we present the results of a numerical analysis of our model. We assume that the economy is in a steady state. We use the following CES utility function and Cobb-Douglas production function:

\[
U(c_1, c_2, h) = \frac{c_1^{1-\alpha}}{1-\alpha} + \frac{c_2^{1-\alpha}}{1-\alpha} + \frac{h^{1-\sigma}}{1-\sigma},
\]

\[
F(l^F, l^I) = (l^F)^\kappa (l^I)^{1-\kappa}.
\]

We investigate numerically the optimal monetary policy and the optimal fiscal and audit policy when the cost of auditing \(K\) varies. The parameters for our numerical exercise are chosen as follows: \(\beta = 0.96, \alpha = 0.7, \sigma = 0.5, \kappa = 0.9, g = 0.1, \) and \(\rho \in [0, 1], \) where \(\rho_M = 1.\)

Table 1 reports some of our findings. When the cost of audit is zero it is optimal to audit everybody since it would eliminate imperfections of the tax instrument. As agents face the same tax rate across sectors, i.e. \(\rho^* = \tau^F\), they are indifferent between working in either sector. And in accordance with Proposition 3, the Friedman rule is optimal in this case. When audit cost is positive, i.e. \(K > 0\), consumption is higher when the agent is not audited, and the agent consumes more of the credit good than the cash good. Notice that the difference between \(c_2^F(\alpha)\) and \(c_2^F(n)\) never disappears because there is an income difference between an audited agent and an unaudited one. In the limit as \(\theta \to 0\), what the auditor collects approaches the tax collection in the formal sector, which does not converge to zero. Thus, there will be a difference between \(U(a)\) and \(U(n)\) that never disappears as \(\theta \to 0\). We observe the same feature when \(\theta \to 1\).

When the cost of audit is positive, it is not optimal to audit everybody. In fact, the audit rate \(\theta\) declines and the penalty rate \(\rho\) increases as the audit cost rises. As a result, the tax rate of the formal sector increases too. The Friedman rule is not optimal when the audit cost is positive. The higher the audit cost the lower the welfare level. It is a direct consequence of the fact that the audit cost enters the resource constraint. As the audit cost increases, consumption and formal
labor decline, and informal labor increases.

Table 1 - Monetary and Fiscal Policy and Allocations

Figure 1 depicts the behavior of the gross nominal interest rate $R$ for various levels of audit rate $\theta$ and penalty rate $\rho$ when the cost of audit is zero. As discussed above, $R > 1$ for small $\theta$. There is a $(\theta, \rho)$ frontier along which the interest rate hits the Friedman rule level of 1 and stays there for higher levels of audit and penalty rates. In our notation, it is the line $(\hat{\theta}(\rho), \rho)$ (see Proposition 5). The optimal combination $(\theta^*, \rho^*)$ lies on this frontier (the arrow indicating $(\theta^*, \rho^*)$ in Figure 1).

Figure 1 - Interest Rate, Audit Rate and Evasion Penalty - $K = 0$

Similarly, Figure 2 shows how the interest rate $R$ depends on $\theta$ and $\rho$ when the cost of audit $K = 0.009$. The Friedman rule is not optimal in this case: $R^* = 1.0407$ with $(\theta^*, \rho^*) = (0.015, 1)$. For the Friedman rule to be restored the audit policy should be brought to the $(\hat{\theta}(\rho), \rho)$ frontier at some welfare loss.

Figure 2 - Interest Rate, Audit Rate and Evasion Penalty - $K = 0.01$

Both Figures 1 and 2 indicate that for a fixed evasion penalty rate $\rho$, the interest rate $R$ is a decreasing function of the audit rate $\theta$. And $R$ reaches the Friedman rule level only when $\theta$ is high enough. If, in similar fashion, we fix the audit rate $\theta$ then $R$ is a decreasing function of $\rho$ and reaches the Friedman rule level of 1 only when $\rho$ is high enough. To the extent that $\theta$ and $\rho$ can be considered as tax instruments for the informal sector, this observed behavior of the interest
rate suggests that the Friedman rule is attained when the informal sector is taxed at a sufficiently high level (even though this taxation is imperfect).

The graphs in Figure 3 exhibit the behavior of the optimal levels of auditing rate $\theta$, penalty rate $\rho$, interest rate $R$, and welfare $U$ as functions of the audit cost $K$. As can be seen in the first panel, the optimal audit rate is one when the cost $K = 0$, but quickly falls and becomes zero starting at $K = 0.01$. The optimal penalty rate, on the other hand, rises from the level of 0.2 to 1. Note that here we assume that the evasion penalty ranges from zero to one, i.e. $\rho \in [0, 1]$, meaning that the maximal penalty level that the government can impose on a tax evader is one hundred percent of the tax evaded. For any cost of audit sufficiently high, it is optimal to impose the highest evasion penalty available, in our numerical exercise $\rho = 1$. The rising nominal interest rate curve and declining welfare curve confirm our observations from Table 1. That is, as audit cost increases the optimal interest rate decreases and welfare falls. Both curves stabilize at $K = 0.01$, at which level the audit rate falls to zero.

Figure 3 - Audit Rate, Evasion Penalty, Interest Rate and Welfare vs. Audit Cost

Figure 4 shows that there are welfare losses to recover the Friedman rule when the cost of auditing is different from zero. Moreover, any deviations from the optimal monetary policy imply welfare costs. As expected, welfare cost is higher the higher the audit cost. Although welfare losses are quantitatively small, this result suggests a trade-off between social welfare and the Friedman rule prescription in an economy with informal activities and imperfect taxation where monetary and fiscal policy are chosen optimally. Finally, our numerical results are not sensitive to changes in parameters $\alpha$ or $\sigma$ in the utility function and $\kappa$ in the production function.

Figure 4 - Welfare Cost and Optimal Interest Rate
5 Conclusion

This paper studies the optimal monetary and fiscal policy in the presence of informal activities and tax evasion when the government has the tools needed to deal with informal activities and can choose them optimally to reduce fiscal distortions. Taxation of informal labor is imperfect. When auditing is costless, the Friedman rule is the optimal monetary policy, which is attained when all agents are audited and informal and formal labor are taxed at the same rate, as efficiency requires. In the presence of auditing costs, the optimal audit rate and evasion penalty do not implement the Friedman rule, and the optimal monetary policy is to deviate from it. We observe an inverse relationship between the optimal audit rate and the optimal evasion penalty, a result consistent with an extensive literature on tax evasion and enforcement. The optimal nominal interest rate is decreasing in the audit rate and audit policy parameters play an important role in determining optimal monetary policy. When informal activities are only imperfectly taxed and auditing is costly, there are welfare costs of implementing the Friedman rule prescription.

References


### Table 1 - Monetary and Fiscal Policy and Allocations

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<tr>
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<th>Cost of auditing</th>
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<tr>
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<td>( K = 0 )</td>
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<tr>
<td>Audit rate ( \theta^* )</td>
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<tr>
<td>Penalty rate ( \rho^* )</td>
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<tr>
<td>Tax rate ( \tau^{F*} )</td>
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<tr>
<td>Interest rate ( R^* )</td>
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<tr>
<td>Welfare ( U^* )</td>
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<td>Cash consumption ( c^*_1 )</td>
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<td>Informal labor ( l^{I*} )</td>
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**Figure 1** - Interest Rate, Audit Rate and Evasion Penalty - \( K = 0 \)
Figure 2 - Interest Rate, Audit Rate and Evasion Penalty - $K = 0.01$

Figure 3 - Audit Rate, Evasion Penalty, Interest Rate and Welfare vs. Audit Cost
Figure 4 - Welfare Cost and Optimal Interest Rate