Informal Work Networks

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September 12, 2008

Abstract

In this paper, we present a model of time allocation to formal and informal work, where the informational structure of the informal sector leads to peer effects in labor provision, and this effect is stronger in weaker institutional settings. In our model workers allocate their time between formal and informal labor markets. The formal sector is competitive and subject to taxation, while informal jobs are not taxed, but are subject to informational frictions. The informal labor market is driven by the social networks of workers. Informal time allocation is increasing in the strength of social ties; Because workers who already have informal work may pass job information to their peers, formal labor supply is decreasing in the number of peers with an informal job, while informal labor supply is increasing. Weaker institutions lead workers to allocate more time to the informal sector, but this may result in lower income. In any environment, stronger social ties improve the transmission of job information and increases income.

Keywords: Informal Activities, Social Networks, Labor Markets

JEL Classification: E26, D85, J22, J64

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1 Introduction

In this paper we present a model of time allocation to formal and informal work, where the informational structure of the informal sector leads to peer effects in labor provision, and this effect is stronger in weaker institutional settings. In our model workers allocate their time between formal and informal labor markets. The formal sector is competitive and subject to taxation, while informal jobs are not taxed, but are subject to informational frictions. The informal labor market is driven by the social networks of workers. The informational structure of the informal market follows Calvó-Armengol and Jackson [10].

Workers learn about informal job opportunities, and may pass this information to their peers. We characterize the equilibrium time allocations and derive formal and informal labor provision as functions of taxation, enforcement, and the properties of the social network. Workers that are well connected to their peers, and workers with many peers engaged in informal work, are themselves more likely to engage in informal activities. This suggests that even accounting for differences in taxation and enforcement, differences in social structures leads to differences in informal labor provision. This effect is stronger when institutions are weaker.

This model allows for the traditional explanations of informal work, i.e., taxation and tax enforcement. Time allocated to informal work is decreasing in the detection probability and increasing in the tax rate on formal labor, as expected. Workers who already have an informal job do not take social ties into account when choosing informal labor and they allocate more time to informal work than a worker who does not. For formal workers, however, informal time allocation is increasing in the probability that they receive informal job information from one of their peers. Because workers who already have informal work may pass job information to their peers, formal labor supply is decreasing in the number of peers with an informal job, while informal labor supply is increasing.

We model weaker intuitions by a lower evasion detection probability, and workers are more likely to keep their informal jobs. This may be due to corruption or the lack appropriate of instruments to enforce tax obligations. In this environment informal work is more attractive, and peer effects are stronger. Workers’ time allocation becomes more sensitive to the possibility of receiving job information from their peers, and their chances of finding informal employment rise much more from stronger social ties, than when institutions are stronger. However, this does not necessarily lead to higher income. Weaker institutions lead workers to allocate more time to the informal sector, but this may result in lower income. On the other hand, for any environment, stronger social ties improve the transmission of job information and increases income.
It has long been understood that social ties are important in labor markets (see Granovetter [23], and Ionnides and Loury [24] for a recent survey). Numerous studies have shown the importance of referrals and word of mouth for job search (Bradshaw [8], Bortnick and Ports [7], Blau [5], Blau and Robins [6]). Such referrals come from a worker’s peers, family, and co-workers. The nature of this social network can have a large effect on job search (see, for instance, Minshi [27] for a study of Mexican immigrants in the United States). It is also clear that usage of social networks varies along a variety of dimensions, such as gender, age, education, ethnicity and occupation (see Bradshaw [8], Ports [29], Elliott [17]). While many different forces might lead to differential usage of social networks, we focus in this paper on differences that are the result of frictions in job search created by the different legal status of formal and informal labor.

The importance of social connections for job search varies by occupation (see Ionnides and Loury [24]), and there is evidence that in the informal sector, social capital is especially important. For example, Rees [30] found that 80 percent of blue collar jobs were found using social contacts, as opposed to 50 percent of white collar jobs. This difference will be even more pronounced for jobs in the informal sector. Rees [30] also made the distinction between the different sources of information workers use when searching for jobs. These can be broadly classed as formal sources of information, such as job postings and placement agencies, and informal sources of information, such as family, peers and coworkers. We interpret this asymmetry between the two sectors in our model to be the result of legal frictions and the nature of informal activities that prevent firms and workers from finding one another. Firms and workers must instead rely on word of mouth, referrals and happenstance to find each other. This process will depend on the social network of contacts, colleagues, friends and family of the worker. It is natural, therefore, to model the market for informal work differently than the market for formal work.

This paper also contributes to the recent renewed interest in informal activities more generally. We define informal activities as all income generating activities which do not comply with tax obligations, i.e., tax evasion and non-compliance with economic legislation. This kind of labor may take many different forms. For instance, Schneider and Enste [32] argue that the informal use of labor may consist of a second job after (or even during) regular working hours. Pedersen [28] provides evidence that, in Denmark, Norway, Sweden, Germany and the United Kingdom, employment in the formal sector determines whether people carry out informal activities, suggesting that workers build up an informal network of contacts in their formal job, and borrow a workshop for informal activities on the weekend or after-hours. Thus, while the formal job is obtained through public sources of information, informal activities require social contacts to carry out. It
is just such a situation we capture in this paper.

Other studies that consider workers’s decisions to engage in informal work are largely based on a single, binary decision of whether to engage in informal activities (see Allingham and Sandmo, [1], Cremer and Gahvari, [14]; Lemieux et al., [26]; Slemrod and Yitzhaki, [33], Sandmo, [31] and Davidson et al. [18]). These papers do not explicitly consider the informational structure of the informal job market or peer effects. Several works have studied the effects of government interventions, such as taxation and labor market regulations (Banerjee and Newman, [4]; Johnson et al., [25], Friedman et al., [19]; Schneider and Enste [32]; Fugazza and Jacques, [21]), and the impact of bureaucracy, corruption and other institutional and enforcement conditions on informal labor (Busato and Chiarini, [9]; Choi and Thum, [12]; Dabla-Norris et al., [15]) and income inequality (Chong and Gradstein, [13]). Other studies have argued that the heterogeneity of firms and entrepreneurs and limited access to capital markets are key to explain the emergence of informal activities (Dessy and Pallage, [16]; Gordon and Li, [22]; Amaral and Quintin, [2]; Antunes and Cavalcanti, [3]). We introduce a very specific kind of heterogeneity among workers, both in the transmission of job information from one worker to another, and heterogeneity over time in the employment status of each worker’s peers. These two peer effects drive our results¹.

The paper proceeds as follows: In section 2 we describe the basic model, and we describe its equilibria in section 3. In section 4, we illustrate our main results and the features of the model with a numerical exercise, and section 5 concludes.

2 The Model

Let \( N = \{1, \ldots, n\} \) be the set of workers. Each worker has a utility function \( u(c, h) \), where \( c \) is consumption and \( h \) is leisure. They may choose to work in the labor market, which has two sectors, formal and informal. Each worker \( i \) must choose how much time to allocate to work in the formal sector, \( l_i \), and how much time to allocate to work in the informal sector, \( \gamma_i \). Time allocated to informal sector may not necessarily be spent in work. If the worker is not matched to an informal job, he may unwillingly spend it on leisure. We track a worker’s informal employment status by a state variable \( S_{it} \), where \( S_{it} = 1 \) for an worker \( i \) employed in the informal sector and \( S_{it} = 0 \) for a worker without an informal job.

We assume an informational asymmetry between the two markets. Job information

¹Fortin et al. [20] estimate the impact of social interactions on tax evasion based on the results of a laboratory experiment. The study suggests that workers who believe they are treated unfairly by the tax system are more likely to evade taxes to restore equity.
is transmitted very differently in each sector. The formal labor market is competitive, so that workers can always find a formal job if they want one. The informal market, on the other hand, is mediated by social networks. A worker must hear of an informal job from his peers. Information about formal jobs is public and abundant, while information about informal jobs is only passed from one person to another, i.e., workers learn about informal job opportunities from their peers. If a worker hears about a job, he will either take it himself, if he doesn’t already have an informal job, or he will pass the job to one of his peers.

The informal job transmission process is described by a function $p_{ij} : \mathbb{R}^n \to [0, 1]$, where $p_{ij}(S_t)$ gives the probability that a job originally heard by worker $j$ is eventually received by worker $i$ when the state is $S_t$. In general, we can think of many influences determining the social ties between two workers. Ethnicity, gender or occupation may play important roles. This model is flexible enough to accommodate all of these situations, depending on the application.

We will assume that $p_{ij}(S_t)$ is nondecreasing in $S_t$, i.e., the more workers have informal jobs, the more likely they are to pass information about jobs. This might be, for example, because they pass jobs they themselves do not need. For example, the function

$$p_{ij} = \begin{cases} (1 - S_{i,t-1})\alpha, & i = j \\ S_{j,t-1}\alpha \frac{n_j}{\sum_{k \neq i} g_{jk}}, & i \neq j \\ 0, & \text{otherwise.} \end{cases}$$

describes the case where a worker $i$ hears about an informal job himself with probability $\alpha$ and is passed information from another worker $j$ if $j$ already has an informal job, hears about one, and then passes him that information. The probability $j$ does so is given by the relative strength of the social ties between $i$ and $j$, given by a function $g_{ij}$. This is the transmission function considered in Calvó-Armengol and Jackson [10]. Note that job information is only passed “one step” along the job contact network in this example. Information that is not used by its first recipient is lost. This is a simple kind of social network with peer effects, where the strength of the social ties between $i$ and $j$ is constant, and the prevalence of informal jobs is given by $\alpha$.

If the strength social ties between workers $i$ and $j$ depends on the amount of formal
work $i$ and $j$ engage in, as suggested by Pedersen [28], we have

$$p_{ij} = \begin{cases} 
(1 - S_{i,t-1})\alpha, & i = j \\
S_{j,t-1}\alpha \frac{g_{ij}(l_i,l_j)}{\sum_{k=1}^{N} g_{jk}(l_i,l_j)}, & i \neq j \\
0, & \text{otherwise.}
\end{cases}$$

This is the case where workers work together in the formal market, and provide information to one another about job opportunities in the informal market. In order to have access to informal job, workers must engage in a certain amount of formal work. Thus, the social tie function $g_{ij}$ is a function of formal labor, $l_i$ and $l_j$.

We assume that there are no technological differences between the two sectors of the economy. The only difference is their informational structure and how the worker’s labor is taxed. For this reason, we assume that, except for taxes, formal and informal jobs pay the same wage. The reason for this is to highlight the role of peer effects in time allocation between formal and informal labor markets. Differences in wages between the two sectors lead to differences in time allocation, and could easily be included in the model. This only clouds the impact of social networks, however. We do not explore any role of the government in our economy and the proceeds from taxation and enforcement are dissipated. There is no saving in this economy.

If workers choose to work in the informal sector, they face some chance of detection by the tax authorities. We model this by a parameter $\beta$, which gives the probability of detection, determined by audit policies and monitoring procedures. In the event of detection, the worker loses his informal job, and all income earned that period from informal work is seized.\(^2\) This parameter can also be interpreted as the informal job break-up probability.

Workers face the following budget constraint:

$$c_{it} \leq l_{it}(1 - \tau) + S_{it}(1 - \beta)\gamma_{it}, \quad (1)$$

where consumption is produced in the formal and informal sector according to the constant returns to scale production function $y = \sum_i (l_{it} + S_{it}\gamma_{it})$.

The sequence of events in a given period $t$ is summarized in Table 1. The time allocation decision is made at the beginning of period $t$ and cannot be revised. Workers may hear of new informal jobs, which they will either take themselves or pass to their

\(^2\)While this is an extreme punishment, a more complicated treatment of punishment would not affect any of our results.
peers. Workers who either started the period with an informal job, or obtained one, then
carry their work in the informal and formal sectors. Informal jobs are then detected with
probability $\beta$. Finally, workers consume their income.

Table 1: The sequence of events in period $t$

<table>
<thead>
<tr>
<th>Event</th>
</tr>
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<tbody>
<tr>
<td>0</td>
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<tr>
<td>1</td>
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<td>3</td>
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<tr>
<td>4</td>
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<td>5</td>
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<tr>
<td>6</td>
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<tr>
<td>7</td>
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</tbody>
</table>

The number of informal jobs worker $i$ hears about in period $t$ is $O_{it}$. If $i$ is unemployed in
the informal sector at the start of period $t$, $S_{t-1} = 0$, he will become employed if $O_{it} > 0$. Informal jobs are detected with probability $\beta$ and this includes jobs that just formed. This gives the law of motion of each worker’s employment status in the informal sector:

$$S_{t-1} = 1 \Rightarrow S_t = \begin{cases} 1, & \text{with probability } 1 - b; \\ 0, & \text{with probability } b. \end{cases}$$

$$S_{t-1} = 0 \Rightarrow S_t = \begin{cases} 1, & \text{if } O_{it} > 0, \text{with probability } 1 - b; \\ 0, & \text{otherwise}. \end{cases}$$

3 Analysis

An equilibrium is a profile of time allocation choices $\{l_i, \gamma_i\}_{i=1}^N$ such that, given $l_{-i}$ the
profile of actions of workers besides $i$, and the state of nature $S_t$, $l_i$ maximize expected utility, for every worker $i$. The profile of labor choices must form a static Nash equilibrium of the stage game.

Let $S_t = \{S_{it}\}_{i=1}^n$ be the vector of employment states. We now write the worker’s
dynamic problem as a function of this state $S$.

$$V(S_{t-1}) = \max_{c_{it}, l_{it}, \gamma_{it}} \mathbb{E}U(c_{it}, (1-l_{it} - S_{it}\gamma_{it}) + \delta\mathbb{E}[V(S_t)]$$

subject to

$$l_{it} + \gamma_{it} + h_{it} = 1 \quad (2)$$

$$c_{it} \leq l_{it}(1-\tau) + S_{it}(1-\beta)\gamma_{it}$$

$$\sum c_{it} = \left(\sum (l_{it}(1-\tau) + S_{it}\gamma_{it})\right)$$

$$S_t = G(S_{t-1}, l_{it}, \gamma_{it}, l_{-it}, \gamma_{-it}),$$

where $G$ is the law of motion for $S_t$.

We distinguish between the choices of a worker who already has an informal job and one who does not. We call the former an “informal worker,” and use subscript $E,$ and the latter a “formal worker,” and use subscript $U.$ We now characterize a stationary equilibrium of the model. The utility of an informal worker when he chooses formal labor $l$ and informal labor $\gamma$ is defined as

$$U_E(l, \gamma) = U(c_E, 1-l - \gamma),$$

and similarly, the utility of a formal worker is given by

$$U_U(l) = U(c_U, 1-l),$$

where, according to equation (1), the consumption of informal ($c_E$) and formal workers ($c_U$) are, respectively

$$c_E = l(1-\tau) + \gamma,$$

$$c_U = l(1-\tau).$$

3.1 The Informal Worker Problem

The optimal time allocation of an informal worker is not influenced by his social ties and peers effects. Because he already has an informal job, he does not care about social ties and social connections.
Proposition 1 The equilibrium time allocation \((l_E, \gamma_E)\) choice of an informal worker solves

\[
\max_{l, \gamma} \beta U_U(l) + (1 - \beta) U_E(l, \gamma),
\]

which has first order conditions

\[
\begin{align*}
\beta \frac{\partial U_U(l)}{\partial l} + (1 - \beta) \frac{\partial U_E(l, \gamma)}{\partial l} &= 0, \\
(1 - \beta) \frac{\partial U_E(l, \gamma)}{\partial \gamma} &= 0.
\end{align*}
\]

This choice does not depend on \(p_{ij}\), and so a worker with an informal job is unaffected by his peers. All an informal worker is concerned with is the possibility of detection, \(\beta\), and the tax rate, \(\tau\).

3.2 The Formal Worker Problem

The time allocation of a formal worker, by contrast, will depend on the employment status of his peers, and the social tie function \(p_{ij}\). His choices will exhibit peer effects. Workers without informal jobs are in competition for the job information that their peers may pass them.

The strength of social ties among workers determines the probability their peers pass informal job information along. The probability that worker \(i\) receives at least one offer is \(K = 1 - \prod_j (1 - p_{ij})\). The probability that a worker unemployed in the informal sector finds an informal job and is not detected is \((1 - \beta)K\).

Proposition 2 The equilibrium time allocation \((l_U, \gamma_U)\) of a formal worker solves

\[
\max_{l, \gamma} [(1 - \beta)K] U_E(l, \gamma) + (1 - [(1 - \beta)K]) U_U(l)
\]

with the additional requirement of symmetry. First order conditions are

\[
\begin{align*}
(1 - \beta) \frac{\partial K}{\partial l} (U_E(l, \gamma) - U_U(l)) + (1 - \beta)K \frac{\partial U_E(l, \gamma)}{\partial l} + (1 - (1 - \beta)K) \frac{\partial U_U(l)}{\partial l} &= 0, \\
(1 - \beta)K \frac{\partial U_E(l, \gamma)}{\partial \gamma} &= 0.
\end{align*}
\]

Depending on the social tie function, \(K\) may depend on the formal labor choices of each worker, the number of peers he has, and the strength of social ties between workers, in which case \(\partial K/\partial l\) may be nonzero. In this case, the choice of formal labor affect not
only their income, and leisure, but also the probability they find an informal job. In our earlier example, $K$ is given by

$$K = 1 - (1 - \alpha) \prod_{j \neq i} \left( 1 - S_{jt-1} \alpha \sum_{k \in S_{jt-1}} g_{jk} \right).$$

In some examples we consider, the network structures (and thus, the $p_{ij}$) are unaffected by time allocation choices, that is, social ties are exogenous to the labor choice. The empty network is one extreme case, where no information transmission is possible. For this network, $p_{ij} = 0$ for every pair of workers $ij$, where $i \neq j$. The probability a worker becomes employed in the informal sector is simply $p_{ii}$, the probability he hears of a job himself. In our example, this is the case where $g_{ij} = 0$ for all pairs of workers $ij$, and all social ties have strength zero. That is, workers are not connected to one another. Therefore,

$$K^*_{\text{Empty}} = 1 - (1 - \alpha) \prod_{j \neq i} (1 - 0) = \alpha.$$

On the other extreme, if $g_{ij} = g$ for all pairs of workers, social ties are of equal strength between everyone. This network has the most information transmission. If $m$ is the number of peers worker $i$ has with informal jobs, we have

$$p_{ij} = p^*_{\text{Complete}} = \begin{cases} 
\alpha, & i = j; \\
S_{jt-1} \frac{\alpha}{n-m-1}, & i \neq j.
\end{cases}$$

The probability he receives information from an employed peer (a peer with an informal job) depends on the number of competitors he has for that information, $n - m - 1$. Therefore,

$$K^*_{\text{Complete}} = 1 - (1 - \alpha) \left( 1 - \frac{\alpha}{n-m-1} \right)^m,$$

for every worker without an informal job. Information may be passed from up to $m$ other workers.

Finally, we also consider the case where $p_{ij}$ and $K$ depend on the formal labor choices of each worker. This will be reflected in optimal time allocation of a formal worker. Because the equilibrium is symmetric, it will not change the equilibrium probabilities $p^*$
and $K^*$:

$$p_{\text{Complete}}^*(l_U) = \begin{cases} 
\alpha \frac{g(l_U(m), l_E)}{(n-m-1)g(l_E, l_U(m)) + g(l_U(m), l_E)} & \text{if } i \text{ has an informal job;} \\
0, & \text{otherwise.}
\end{cases}$$

This is the same equilibrium probability as in the case where social ties are exogenous. State transition probabilities will therefore be the same in both cases. This is due to symmetry required in equilibrium.

### 3.3 Comparative Statics

We now examine the comparative statics of the equilibrium labor choices\(^3\). First, the time allocated to informal work by both formal and informal workers is decreasing in the detection probability $\beta$, and increasing in the tax rate on formal labor $\tau$. As expected, increasing the probability workers are caught evading taxes, or decreasing those taxes, will discourage informal work. Second, workers who start the period with an informal job in hand allocate more time to informal work than a worker who does not. Because they are sure to be able to engage in informal work, they set aside more time to do so.

More interesting is the effect of the job transmission function $p$. The informal time allocation of informal workers is independent of $p$. Workers who already have an informal job do not take social ties into account when choosing how much to work in the informal sector. They already have an informal job, and don’t need to rely on their social ties to find one. For formal workers, however, informal time allocation is increasing in $p_{ij}$, the probability that a job originally heard by $j$ is eventually received by $i$.

The effect of changes in $p_{jk}$ is more ambiguous. The function $p$ may reflect the possibility that job information is lost. For instance, in our example, job information is only passed one step along the social network, and so if a worker learns of a job but has no peers without an informal job, to whom he may pass it, that job opportunity is lost, even if there are unemployed workers farther away in the social network. An increase in $p_{jk}$ that reduces lost job information will therefore have two effects. It may increase the competition a worker faces for job information, which leads him to decrease the time he allocates to informal work. On the other hand, it will increase the long-run employment status of his peers.

In the long run, employment in the informal sector among workers is positively correlated. That is, holding constant the job transmission function $p$, the probability that

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\(^3\)See Appendix for details.
at any particular period worker $i$ has an informal job is positively correlated with probability one of his peers has an informal job. Therefore, an increase in $p_{jk}$ that increases the probability a competitor for $j$’s information receives that information, without actually reducing the probability that worker $i$ receives it, will in the long run increase $i$’s probability of being employed in the informal sector. An increase in $p_{jk}$ that comes at the expense of $i$ will not have this effect. It will decrease his time allocation to informal work, without any increase in his long run employment status.

Our model thus illustrates the importance of peer effects in worker’s decision to work in the informal sector. When job information on informal work passes from worker to workers, through a social network, then peer effects will influence time allocation.

In our previous example, the job transmission function $p$ depends on a job arrival probability $\alpha$ and the strength of social ties between workers, $g_{ij}$. The comparative static effects of changes in $\alpha$ and $g$ can be discerned by their effect on $p_{ij}$. A rise in $\alpha$ will increase $p_{ij}$, and so increase time allocated to the informal sector. A rise in $g_{ij}$ will increase $p_{ij}$, but a rise in $g_{kj}$ will decrease $p_{ij}$. If the social ties between $i$ and $j$ become stronger, information is more likely to be passed from to $i$ from $j$. If another worker $k$’s ties to $j$ become stronger, then because $i$ and $k$ are competitors for $j$’s information, $p_{ij}$ decreases, and so time allocated to informal work falls.

4 Informal Work Network: An Example

In this section we will compare time allocation for the particular job transmission function described in section 2. The impact of peer effects is especially easy to see in this example, by considering different forms of the social tie function $g_{ij}$. We solve the model numerically to find the optimal time allocation to the formal and informal sectors, and illustrate the dynamics of the economy with a simulation. This allows us to evaluate the quantitative implications of network structures for time allocation and income. We demonstrate that peer effects are stronger in weaker institutional environments, and that weakening the enforcement of tax obligations, while increasing the benefit a worker receives from informal work, can actually lower his income.

We first consider a baseline economy where the formal tax rate $\tau$, job arrival probability $\alpha$ and detection probability $\beta$ for this example are $\tau = 0.25$, $\alpha = 0.10$ and $\beta = 0.70$, respectively. These values, to some extent, resemble a developed economy, with strong enforcement of labor laws. We assume $n = 3$, so that we are considering the simplest
non-trivial network. Recall that the job transmission function \( p_{ij} \) has the following form:

\[
p_{ij} = \begin{cases} 
(1 - S_{i,t-1})^\alpha, & i = j \\
S_{j,t-1}^\alpha \sum_{k} \frac{g_{ij}}{g_{j,k}}, & i \neq j \\
0, & \text{otherwise.}
\end{cases}
\]

Differences in \( g_{ij} \) describe differences in the strength of the social ties between workers, and we will examine three cases. First, when \( g_{ij} = 0 \) for all workers, the social network is empty, there are no social connections, and thus no peers effects. In this case workers are disconnected from one another, and cannot learn about job opportunities from each other. Each worker must fend for himself.

Second, when \( g_{ij} = g \) for all workers, the network is complete, and all workers are equally connected. In this case the social ties between all workers are equally strong, and workers will pass job information to one another. Peer effects will be present in this case, because workers have access to the information of their peers.

Finally, if \( g_{ij} = g(l_i, l_j) \) social tie strength is endogenous, and depends on the amount of formal labor workers provide. This is the case, for example, when information passes between coworkers, or if access to capital in the formal market is required to engage in informal work. The social network is complete, but may be asymmetric. The strength of the social tie between two workers will depend on how much they work together. Because of this, there is an incentive to engage in more formal work, in order to strengthen your ties to your peers.

Recall that \( l_E \) is the formal labor supply of a worker employed in an informal job, and \( l_U(m) \) is the formal labor supplied by a worker not employed in an informal job, when he has \( m \) peers employed in the informal sector. We solve the model assuming that the utility function is of the form

\[
u(c, h) = c^{1/2}h^{1/2}
\]

which is a standard specification\(^4\).

The time allocation of an informal worker, \( l_E \) and \( \gamma_E \), and of a formal worker, \( l_U(m) \) and \( \gamma_U(m) \), for our baseline economy is presented in Table 2. In each network structure, informal workers always work less in the formal sector than formal workers. The informal time allocation of a worker employed in the informal sector (\( \gamma_E \)) and of an unemployed worker (\( \gamma_U(0) \)) without a social network (that is, the empty network), represent the

\(^4\)The results that follow are robust to other specifications of the utility function \( u \) and strength of social ties function \( g \).
highest and lowest allocations, respectively. For all other networks, formal and informal
time allocation will be between these two extremes. Since formal workers hope to receive
informal job information from their peers, their time allocation choices show evidence of
peer effects.

Table 2: Baseline Economy - Formal and Informal Labor Supply

<table>
<thead>
<tr>
<th></th>
<th>Empty</th>
<th>Complete Endogenous</th>
<th>Complete Exogenous</th>
</tr>
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<tbody>
<tr>
<td>Formal Worker</td>
<td>( l_U(0) ) 0.49809</td>
<td>0.49809</td>
<td>0.49809</td>
</tr>
<tr>
<td></td>
<td>( \gamma_U(0) ) 0.06168</td>
<td>0.06168</td>
<td>0.06168</td>
</tr>
<tr>
<td>Formal Worker</td>
<td>( l_U(1) ) -</td>
<td>0.49723</td>
<td>0.49719</td>
</tr>
<tr>
<td></td>
<td>( \gamma_U(1) ) -</td>
<td>0.06243</td>
<td>0.06246</td>
</tr>
<tr>
<td>Formal Worker</td>
<td>( l_U(2) ) -</td>
<td>0.49456</td>
<td>0.49456</td>
</tr>
<tr>
<td></td>
<td>( \gamma_U(2) ) -</td>
<td>0.06478</td>
<td>0.06478</td>
</tr>
<tr>
<td>Informal Worker</td>
<td>( l_E ) 0.47457</td>
<td>0.47457</td>
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</tr>
<tr>
<td></td>
<td>( \gamma_E ) 0.08237</td>
<td>0.08237</td>
<td>0.08237</td>
</tr>
</tbody>
</table>

If a worker’s peers do not have informal jobs - that is, if the state is \( (m = 0) \) - formal
and informal time allocations are the same in each network structure. Without peers
who may pass you information, every structure is equivalent to the empty network. As
more peers become employed in the informal sector, formal workers allocate less time to
formal work, and more time to informal work. In other words, for formal workers, formal
labor supply is decreasing in the number of peers with an informal job, and conversely,
informal labor supply is increasing in the number of peers with an informal job. If the
number of peers working in the informal sector is small, a formal worker is in competition
with other peers for the information about new informal job opportunities. When social
ties are endogenous, this provides workers an incentive to manipulate their social ties
with their formal labor choice. There is less incentive to do so when you have fewer
competitors for informal job information.

In our economy, \( l_U(2) \) and \( \gamma_U(2) \) describes the labor supply of a formal worker when
everybody else, except him, already has an informal job. In this situation, the structure
of the network plays an interesting role. There is no distinction between endogenous and
exogenous network. Workers have no competitors for any information that arrives, so
they need not manipulate their social ties. In a complete network any informal worker
peers can pass informal job information to the formal worker.

Figures 1 and 2 shows the comparative statics of the equilibrium informal labor al-
location in the complete, exogenous network, and the empty network, respectively. We
start from our baseline values \( (\tau = 0.25, \alpha = 0.10, \beta = 0.70) \) and shows the effect of
changes in each variable in turn. Notice that formal labor is never greater than 0.5, due to
the utility function we use. Different specifications do not change the qualitative effects.
The labor of a worker who has an informal job is insensitive to the arrival probability, $\alpha$, while the signs of the other effects are as expected and presented in section 3.3.

Because the equilibrium state transition probabilities do not depend on time allocation choices, the empty and complete networks each have a state transition matrix that is only a function of $\alpha$, $\beta$ and $n$. We illustrate, for $n = 3$, these transition probabilities, for the empty and the complete network, in figures 3 and 4, respectively. The regions of the unit square, corresponding to different values of $\alpha$ and $\beta$, are colored according to which state is likeliest for those values. State behavior is similar in all networks. If the probability of detection $\beta$ is close to 1, zero informal workers is the most likely state. As $\alpha$ increases, the cutoff at which this state becomes the likeliest increases. For lower values of $\beta$, states with more informal workers are more likely. While the empty network and the complete network appear similar, there are differences in these state probabilities. For example, three informal workers are more likely in the complete network than the empty network for all values of $\alpha$ and $\beta$.

To investigate the effect of different institutions on labor allocation, and their interaction with peer effects, we investigate an alternative economy, with weaker enforcement. We keep the job arrival probability and tax rate as in our baseline economy ($\alpha = 0.10$, $\tau = 0.25$). A lower detection probability ($\beta = 0.30$) captures an environment where enforcement is more lax due to, for instance, corruption, weaker institutions or lack of appropriate policy instruments to fight tax evasion. Table 3 presents time allocations under different network structures for the alternative economy.

Again, an informal worker works less in the formal sector than any other formal work peer. Formal workers allocate less time to formal work, and more to informal work. This is consistent with a weaker enforcement environment. Note also that differences in labor time allocation are larger now. Consider a formal worker with one peer employed in the informal sector. Besides the fact that he works less in the formal sector, he works much
less if there are no peer effects (as in the empty network) than when there are stronger peer effects (as in the exogenous complete network).

This demonstrates that the effects of the network structure and the information transmission mechanism are stronger when institutions are weaker. When the detection probability is low, a worker is more likely to keep his informal job and so he cares relatively more about informal job opportunities. The results presented for the baseline economy do not suggest large differences due to network structures. However, in an economy where enforcement is weaker, social ties matter more and workers alter their behavior in order to influence their chance of getting an informal job. That is, the informal job information transmission process is more important in an environment where informal jobs do not break up easily.

The dynamics of the economy and the importance of peer effects are especially apparent in our simulation. Taking one week to be the unit length of time, and starting from the state where no workers have informal jobs, we follow the evolution of the economy for 2080 periods, or 40 years. It is important to note that because there is no savings in this model, the only interdependence across periods in income and time allocation is due to peer effects. We simulate the baseline and alternative economy, for the empty and complete networks. We present total income, and income from the formal sector in Figure 5, and then present the gap between these, i.e., the informal labor income, in Figure 6.

Table 3: Alternative Economy - Formal and Informal Labor Supply

<table>
<thead>
<tr>
<th></th>
<th>Empty</th>
<th>Complete Endogenous</th>
<th>Complete Exogenous</th>
</tr>
</thead>
<tbody>
<tr>
<td>Formal Worker</td>
<td>$l_U(0)$</td>
<td>0.49433</td>
<td>0.49433</td>
</tr>
<tr>
<td></td>
<td>$\gamma_U(0)$</td>
<td>0.07240</td>
<td>0.07240</td>
</tr>
<tr>
<td>Formal Worker</td>
<td>$l_U(1)$</td>
<td>-</td>
<td>0.49163</td>
</tr>
<tr>
<td></td>
<td>$\gamma_U(1)$</td>
<td>-</td>
<td>0.07474</td>
</tr>
<tr>
<td>Formal Worker</td>
<td>$l_U(2)$</td>
<td>-</td>
<td>0.48248</td>
</tr>
<tr>
<td></td>
<td>$\gamma_U(2)$</td>
<td>-</td>
<td>0.08265</td>
</tr>
<tr>
<td>Informal Worker</td>
<td>$l_E$</td>
<td>0.34727</td>
<td>0.34727</td>
</tr>
<tr>
<td></td>
<td>$\gamma_E$</td>
<td>0.19961</td>
<td>0.19961</td>
</tr>
</tbody>
</table>

Table 4 lists the long run averages of formal, informal, and total income, using the invariant distribution of the model.
Table 4: Average Yearly Income in Simulation

<table>
<thead>
<tr>
<th></th>
<th>Complete Network</th>
<th>Empty Network</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Baseline</td>
<td>Alternative</td>
</tr>
<tr>
<td>Formal Income</td>
<td>19.207</td>
<td>17.9554</td>
</tr>
<tr>
<td>Informal Income</td>
<td>0.940463</td>
<td>1.90117</td>
</tr>
<tr>
<td>Total Income</td>
<td>20.1475</td>
<td>19.8565</td>
</tr>
<tr>
<td>Probability of Informal Employment</td>
<td>0.043</td>
<td>0.223</td>
</tr>
</tbody>
</table>

First, total income and formal income are higher in the baseline economy than in the alternative economy, for either network. This is because in the alternative economy, due to lax enforcement, workers allocate more time to informal labor and less time to formal labor. Because detection is still possible, however, the informal sector is still risky, and the lost income from the formal sector is not fully replaced by work in the informal sector, and total income falls. Thus, a more lax enforcement environment, while associated with more time allocated to a higher earning sector, leads to less income.

Second, total income and informal income are higher in the complete network than in the empty network, for both the baseline and alternative economy. Workers allocate more time to informal work, and less to formal, but because the detection probability remains the same in the two cases, informal work is not riskier, as it is when comparing the baseline and alternative economies. For this reason, the transfer of time from formal to informal work increases income. This effect is larger for the alternative economy than for the baseline economy, indicating that in a lax enforcement environment, greater access to informal job information is more beneficial than in a stricter enforcement environment. This is because in the baseline, there is a relatively small change in a worker’s chances of finding informal work when peer effects become stronger, i.e., a 4.3% chance versus a 4.1% chance. In the alternative consonancy, due to weak enforcement, informal work is much easier to find when peer effects are stronger, a 22.3% chance versus a 18.9% chance.

These simulations suggest that while lax enforcement and better job information transmission both lead workers to allocate more time to the informal sector, they will have a different effect on the worker’s ex-post income. More lax enforcement is associated with lower income, while better job information transmission is associated with higher income, and the institutional environment determines the importance of peer effects.

5 Conclusions

The importance of social ties and social practices to labor market outcomes has been long understood. This paper formally models the effect of network structure on time
allocation and informal work. It is flexible enough to accommodate many different social structures, but simple enough to generate empirical predictions. For instance, this model provides motivation for many observed features of the informal labor market. In our social network model, two well emphasized determinants of informal activities, tax burden and institutional quality, are present, and we can see how peer effects interact with them. An employed informal worker tends to increase his formal labor supply as tax rates decrease, and as enforcement becomes more lax. When enforcement is weaker, social ties matter more and workers are more responsive to the strength of social ties: The informal job information transmission process is more important in an environment with weaker enforcement. Workers allocate more time to informal activities in the presence of lax enforcement and better job information transmission; thus the institutional environment determines the importance of peer effects for labor time allocation. Further analysis of the different network structures may provide insights into different social institutions.
References


Appendix

Proof of Comparative Static Properties of Section 3.3

To see that the amount of formal labor provided by a formal worker is larger than that provided by an informal worker, we examine the first order conditions of a formal worker.

\[
\beta \frac{\partial U_U(l)}{\partial l} + (1 - \beta) \frac{\partial U_E(l, \gamma)}{\partial l} = 0,
\]

while the first order condition of an informal worker is given by

\[
(1 - \beta) \frac{\partial K(l)}{\partial l} (U_E(l, \gamma) - U_U(l)) + (1 - \beta) K(l) \frac{\partial U_E(l, \gamma)}{\partial l} + (1 - (1 - \beta) K(l)) \frac{\partial U_U(l)}{\partial l} = 0.
\]

We first note that \((1 - (1 - \beta) K(l)) \leq \beta\) and \((1 - \beta) \geq (1 - \beta) K(l)\) for all \(l\), so that the probability of being employed in an informal job is always lower for an unemployed worker than for an employed worker. Furthermore, the term \((1 - \beta) \frac{\partial K(l)}{\partial l} (U_E(l, \gamma) - U_U(l))\) is positive. Therefore, in order to satisfy the first order condition of the unemployed worker, \(l_U\) must be larger than \(l_E\); \(\frac{\partial U_E(l, \gamma)}{\partial l}\) is the only negative term in her FOC, and in order to balance \((1 - \beta) \frac{\partial K(l)}{\partial l} (U_E(l, \gamma) - U_U(l))\) and the more weighted term \(\frac{\partial U_U(l)}{\partial l}\), it must be even more negative, that is, \(l_U\) must be even farther from the optimal labor choice when employed in an informal job. This requires that \(l_U\) be larger. ■

To see the impact of \(\tau\) and \(\beta\) on time allocation, we inspect the worker’s first order condition, starting with formal labor allocation:

\[
\beta \frac{\partial U_U(l)}{\partial l} + (1 - \beta) \frac{\partial U_E(l, \gamma)}{\partial l} = 0.
\]

Because we have \(\beta > 0\), it must be that in equilibrium either both \(\frac{\partial U_U(l)}{\partial l}\) and \(\frac{\partial U_E(l, \gamma)}{\partial l}\) are zero, or one is positive and one is negative, in order to satisfy the FOC. It is impossible that they are both zero at the same formal formal labor choice \(l\). Because the marginal utility of labor is decreasing in consumption and increasing in leisure, a worker who is unemployed in an informal job necessarily has a higher marginal utility of labor than an worker who is employed in an informal job. Therefore, in order to satisfy the FOC, we have

\[
\frac{\partial U_U(l)}{\partial l} > 0 > \frac{\partial U_E(l, \gamma)}{\partial l}.
\]
Using the implicit function theorem and the worker’s FOC, we calculate $\frac{\partial l}{\partial \beta}$:

$$\frac{\partial l}{\partial \beta} E = \frac{\partial U_E(l, \gamma)}{\partial l} - \frac{\partial U_U(l)}{\partial l} \beta \frac{\partial^2 U_U(l)}{\partial l^2}.$$

By the argument above, the numerator is negative. Note that the denominator is the second order condition of the worker’s problem, which at the optimal choice is negative. Therefore, $\frac{\partial l}{\partial \beta} E \geq 0$. Rewrite the informal worker’s first order conditions as follows:

$$\beta \frac{\partial U_U(l)}{\partial l} + (1 - \beta) \frac{\partial U_E(l, \gamma)}{\partial l} = \beta[U_{U1}(1 - \tau) + U_{U2}(-1)] + (1 - \beta)[U_{E1}(1 - \tau) + U_{E2}(-1)] = 0.$$

Where $U_{xy}(c, h)$, $x = E, U$ and $y = 1, 2$ denotes the marginal utility of formal ($U$) and informal ($E$) workers. Using the implicit function theorem and the worker’s FOC, we calculate $\frac{\partial l}{\partial \tau}$:

$$\frac{\partial l}{\partial \tau} E = \frac{bU_{U1} + (1 - \beta)U_{E1}}{\beta[U_{U11}(1 - \tau) - U_{U22}] + (1 - \beta)[U_{E11}(1 - \tau) - U_{E22}]}.$$

As discussed before the numerator is positive and the denominator is negative. This implies that $\frac{\partial l}{\partial \tau} E \leq 0$. Regarding the comparative statics of informal work $\gamma$ recall the worker’s first order condition:

$$(1 - \beta) \frac{\partial U_E(l, \gamma)}{\partial \gamma} = (1 - \beta)[U_{E1} + U_{E2}(-1)] = 0.$$

Follow the steps described above, we obtain:

$$\frac{\partial \gamma_E}{\partial \beta} = \frac{\partial U_E(l, \gamma)}{\partial \gamma} \leq 0,$$

and

$$\frac{\partial \gamma_E}{\partial \tau} = \frac{U_{E11} - U_{E21}}{(U_{E11} - U_{E21}) + (U_{E22} - U_{E12})} \geq 0.$$

where $U_{E11}$ and $U_{E22}$ are negative and $U_{E21}$ and $U_{E12}$ are positive. The comparative static properties of a formal (unemployed) worker are quite intuitive and their proofs follows the same steps presented for the informal worker’s properties. Hence, we will focus and discuss those that are more interesting, i.e. $\frac{\partial l_U(m)}{\partial \beta} \geq 0$ and $\frac{\partial l_U(m)}{\partial \alpha} \leq 0$. The objective of
the worker who is unemployed in the informal sector is given by

$$(1 - \beta)K(l)U_E(l, \gamma) + (1 - (1 - \beta)K(l))U_U(l).$$

The first order condition is

$$(1 - \beta)K'(l)(U_E(l, \gamma) - U_U(l)) + (1 - \beta)K(l)\frac{\partial U_E(l, \gamma)}{\partial l} + (1 - (1 - \beta)K(l))\frac{\partial U_U(l)}{\partial l} = 0.$$ 

Define the worker’s second order condition:

$$SOC = (1 - \beta)(\frac{\partial}{\partial l}[K'(l)(U_E(l, \gamma) - U_U(l))] + (1 - \beta)K'(l)(\frac{\partial U_E(l, \gamma)}{\partial l} - \frac{\partial U_U(l)}{\partial l}))$$

$$+ (1 - \beta)K(l)\frac{\partial^2 U_E(l, \gamma)}{\partial l^2} + (1 - (1 - \beta)K(l))\frac{\partial^2 U_U(l)}{\partial l^2}.$$ 

By the implicit function theorem, we can calculate $\frac{\partial l}{\partial \beta}$:

$$\frac{\partial l}{\partial \beta} = \frac{\frac{\partial}{\partial l}[K(l)(U_E(l, \gamma) - U_U(l))]}{SOC}.$$ 

The denominator is the second order condition of the worker’s problem, which is negative at the optimal formal labor choice $l$. The numerator is negative if $\frac{\partial}{\partial l}[K(l)(U_E(l, \gamma) - U_U(l))] < 0$. Let $\Delta(l) = (U_E(l, \gamma) - U_U(l))$. We can write this condition in the following way:

$$\frac{\partial}{\partial l}[K(l)\Delta(l)] < 0$$

$$\frac{\partial K(l)}{\partial l} \Delta(l) + K(l)\frac{\partial \Delta(l)}{\partial l} < 0$$

$$\frac{\partial K(l)}{\partial l} \Delta(l) < -K(l)\frac{\partial \Delta(l)}{\partial l}$$

$$\frac{\partial K(l)}{\partial l} < -\frac{\partial \Delta(l)}{\partial l}$$

$$\frac{\partial K(l)}{\partial l} l < -\frac{\partial \Delta(l)}{\partial l} l$$

That is, the numerator is negative if the elasticity of the probability of receiving an offer, with respect to $l$, is less than the negative elasticity of the difference in utility between being employed and unemployed in the informal sector. Therefore, this results depends
upon the form of the worker’s utility function $u$, and the form of the society’s social tie function $g$. We will assume that it holds. It is a condition that is satisfied in many examples, such as Cobb-Douglas utility and $g = l_i l_j$. This is also illustrated in our numerical example.

To see that the amount of informal labor and worker provides is increasing in the number of peers he has with informal jobs, we need only recall that by assumption, $p_{ij}(S)$ is nondecreasing in $S$; that is, the probability $K$ he receives job information is nondecreasing in the number of peers he has with informal jobs. It follows immediately that he allocates more time to informal work.

The correlation of worker’s employment status comes from the form of the job transmission process. A detailed proof is given in Calvo-Armengol and Jackson [11].
Figure 1: The comparative static effects of $a$, $b$, and $\tau$ on labor allocation for the Complete network, for the baseline economy.
Figure 2: The comparative static effects of $a$, $b$, and $\tau$ on labor allocation for the Empty network, for the baseline economy.
Figure 3: The most likely state for the empty network, for $n = 3$, as function of $a$ and $b$. Three workers with informal jobs is yellow, two workers is blue, and one worker is red.
Figure 4: The most likely state for the complete network, for $n = 3$, as function of $a$ and $b$. Three workers with informal jobs is yellow, two workers is blue, and one worker is red.
Figure 5: Simulations of total income (solid) and formal income (dashed), for forty years.
Figure 6: Simulations of informal income for forty years.