STATISTICAL QUALITY CONTROL AND DESIGN

Content

• Quality
• Control charts
• Process capability
• Acceptance sampling

• Reliability
• Quality costs
• Product liability
• Quality systems
• Benchmarking and auditing

COURSE INTRODUCTION

• Text (Donna C.S. Summers, 3rd Edition)
• Website: overheads, assignments, solutions, computer lab documents, midterm and exam information, and other announcements and information
• Lecture Notes (purchase or download)
• Computer Lab
• Background material: probability, statistical tests
• Grading
  – Assignments
  – Midterm
  – Comprehensive Final
A WORD ABOUT CONDUCT

• Basic principles

1. Every student has the right to learn as well as responsibility not to deprive others of their right to learn.

2. Every student is accountable for his or her own actions.

In order for you to get the most out of this class, please consider the following:

a. Attend all scheduled classes and arrive on time. Late arrivals and early departures are very disruptive and violate the first basic principle.

b. Please do not schedule other activities during this class time. I will try to make class as interesting and informative as possible, but I can't learn the material for you.

c. Please let me know immediately if you have a problem that is preventing you from performing satisfactorily in this class.
CHAPTER 1: QUALITY BASICS

Outline

• Definitions of quality
• The evolution of quality
  – Inspection, quality control, statistical quality control, statistical process control, total quality management
• Processes, variation, specification and tolerance limits

Quality and Education

Business has made progress toward quality over the past several years. But I don't believe we can truly make quality a way of life … until we make quality a part of every student’s education

Edwin Artzt, Chairman and CEO, Proctor & Gamble Co., Quality Progress, October 1992, p. 25
Quality and Competitive Advantage

- Better price
  - The better customers judge the quality of a product, the more they will pay for it
- Lower production cost
  - It is cheaper to do a job right the first time than to do it over
- Faster response
  - A company with quality processes for handling orders, producing products, and delivering them can provide fast response to customer requests

Quality and Competitive Advantage

- Reduced Inventory
  - When the production line runs smoothly with predictable results, inventory levels can be reduced
- Improved competitive position in the marketplace
  - A customer who is satisfied with quality will tell 8 people about it; a dissatisfied customer will tell 22 (A.V. Feigenbaum, Quality Progress, February 1986, p. 27)
Customer-Driven Definitions of Quality

• Conformance to specifications
  – Conformance to advertised level of performance
• Value
  – How well the purpose is served at a particular price.
  – For example, if a $2.00 plastic ballpoint pen lasts for six months, one may feel that the purchase was worth the price.

Customer-Driven Definitions of Quality

• Fitness for use
  – Mechanical feature of a product, convenience of a service, appearance, style, durability, reliability, craftsmanship, serviceability
• Performance
  – The ability to satisfy the stated or implied need, operate without deficiencies and faults
Customer-Driven Definitions of Quality

• Support
  – Financial statements, warranty claims, advertising
• Psychological Impressions
  – Atmosphere, image, aesthetics
  – “Thanks for shopping at Wal-Mart”

The Evolution of Quality

• Interchangeable parts (Eli Whitney, 1798)
  – Standardized production
• Inspection
  – Measuring, examining, testing, or gauging of one or more characteristics of a product or service
  – Determining if the product or service conforms to the established standards
The Evolution of Quality

• Quality control
  – Establishing standards
  – Ensuring conformance to the standards
  – Corrective measures
  – Preventive measures

The Evolution of Quality

• Statistical quality control
  – Shewhart control chart uses the concepts of statistics to detect systematic problems that must be fixed in order to prevent the production of a large number of defective items
  – Acceptance sampling eliminates the need for 100% inspection
The Evolution of Quality

- Statistical process control (SPC)
  - The costs are low if the problems are detected early
  - If a large number of defective products are produced, the scrap or rework costs can be high.
  - Prevention of defects by applying statistical methods to control the process is known as SPC
  - Prevention refers to those activities designed to prevent defects, defectives, and nonconformance in products and services

<table>
<thead>
<tr>
<th>Process</th>
<th>Cost of detection (dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Final testing</td>
<td></td>
</tr>
<tr>
<td>Customer</td>
<td></td>
</tr>
</tbody>
</table>

When defect is detected
The Evolution of Quality

- Total quality management (TQM)
  - A management approach that places emphasis on continuous process or system improvement
  - Based on the participation of all members of an organization to continuously improve the processes
  - Utilizes the strengths and expertise of all the employees as well as the statistical problem-solving and charting methods of SPC
Reading

• Chapter 1:
  – Reading pp. 2-17 (2nd ed.), pp. 2-21 (3rd ed.)

CHAPTER 3: QUALITY IMPROVEMENT

Outline

• Process improvement
  – PDSA cycle
  – Process improvement steps
  – Tools
The P-D-S-A Cycle

Plan
- Identify problem. Develop plan for improvement.

Do
- Implement plan on test basis

Act
- Institutionalize improvement. Continue cycle.

Study
- Is the plan working?

Steps in Process Improvement

- Plan
  1: Recognize problem
  2: Form quality improvement teams
  3: Define problem
  4: Develop performance measures
  5: Analyze problem
  6: Determine possible causes
Steps in Process Improvement

• Do
  7: Implement solution

• Study
  8: Evaluate solution

• Act
  9: Ensure performance
  10: Continuous improvement

Plan: Steps 1 and 2

1: Recognize problem
   – Existence of the problem is outlined
   – In general terms, specifics are not clearly defined
   – Solvability and availability of resources are determined

2: Form quality improvement teams
   – Interdisciplinary
   – Specified time frame
   – Quality circle
Plan: Step 3

3: Define the problem
   – Define the problem and its scope
   – Pareto analysis
   – Brainstorming
   – Why-why diagram

Pareto Chart
A Pareto Chart is used to understand the problem and prioritize the causes of poor quality. It does not locate a solution but helps in identifying the process that leads to the original problem.

For example, a mail-order company might have a goal of reducing the amount of time a customer has to wait in order to place an order. Creating a why-why diagram about waiting on the telephone can help in identifying the root causes.
Why-Why Diagram

Waiting on the phone to place an order

Why?

Insufficient operators available

Why?

Many customers calling at the same time

Why?

All catalogs shipped at the same time

Workers not scheduled at peak times

Low pay

Plan: Step 4

4: Develop performance measures
   – Set some measurable goals which will indicate solution of the problem
   – Some financial measures: costs, return on investment, value added, asset utilization
   – Some customer-oriented measures: response times, delivery times, product or service functionality, price
   – Some organization-oriented measures: employee retention, productivity, information system capabilities
Plan: Steps 5 and 6

5: Analyze problem
   – List all the steps involved in the existing process and identify potential constraints and opportunities of improvement
   – Flowchart

6: Determine possible causes
   – Determines potential causes of the problem
   – Cause and effect diagrams, check sheets, histograms, scatter diagrams, control charts, run charts

Flowchart

- Operation
- Delay
- Storage
- Transportation
- Inspection
- Decision
Enter emergency room

**Flowchart**

- Fill out patient history
- Walk to triage room
- Nurse inspects injury
- Return to waiting room
- Wait for ER bed
- Walk to ER bed
- Walk to Radiology
- Doctor inspects injury
- Wait for doctor
- Walk to radiology
- Technician X-rays patient
- Return to ER bed
- Wait for doctor to return
- Doctor provides diagnosis
- Return to Waiting
- Leave Building
- Pickup prescription
- Walk to pharmacy
- Checkout
Cause and Effect Diagram

- Common categories of problems in manufacturing
  - 5 M’s and an E
    - Machines, methods, materials, men/women, measurement and environment
- Common categories of problems in service
  - 3 P’s and an E
    - Procedures, policies, people and equipment
Check Sheet

COMPONENTS REPLACED BY LAB
TIME PERIOD: 22 Feb to 27 Feb 1998
REPAIR TECHNICIAN: Bob

<table>
<thead>
<tr>
<th>TV SET MODEL 1013</th>
</tr>
</thead>
<tbody>
<tr>
<td>Integrated Circuits</td>
</tr>
<tr>
<td>Capacitors</td>
</tr>
<tr>
<td>Resistors</td>
</tr>
<tr>
<td>Transformers</td>
</tr>
<tr>
<td>Commands</td>
</tr>
<tr>
<td>CRT</td>
</tr>
</tbody>
</table>

![Histogram](histogram.png)

**Histogram**

- **Frequency of calls**
- **Telephone call duration, min**
Scatter Diagram

Rotor speed, rpm

Number of tears

Control Chart

Sample number

Number of defects

UCL = 23.35

$c = 12.67$

LCL = 1.99
Do: Step 7

7: Implement the solution

– The solution should
  • prevent a recurrence of the problem
  • address the root cause of the problem
  • be cost effective
  • be implemented within a reasonable amount of time
– Force-field analysis

Do: Step 7

– Force-field analysis
  • A chart that lists
    – the positive or driving forces that encourages improvement as well as
    – the restraining forces that hinders improvement
  – Actions necessary for improvement
Do: Step 7

Example: create a force-field diagram for the following problem:
- Bicycles are being stolen at a local campus. Campus security is considering changes in the bike rack design, bike parking restrictions and bike registration to try to reduce thefts. Thieves have been using hacksaws and bolt cutters to remove locks from the bikes.

Reading

- Chapter 3:
  - Reading pp. 64-97 (2nd ed.), pp. 52-102 (3rd ed.)
CHAPTER 5: VARIABLE CONTROL CHARTS

Outline

• Construction of variable control charts
• Some statistical tests
• Economic design

Control Charts

• Take periodic samples from a process
• Plot the sample points on a control chart
• Determine if the process is within limits
• Correct the process before defects occur
Types of Data

- Variable data
  - Product characteristic that can be measured
    - Length, size, weight, height, time, velocity

- Attribute data
  - Product characteristic evaluated with a discrete choice
    - Good/bad, yes/no

Process Control Chart

Upper control limit

Process average

Lower control limit

Sample number
Variation

- Several types of variation are tracked with statistical methods. These include:
  1. Within piece variation
  2. Piece-to-piece variation (at the same time)
  3. Time-to-time variation

Common Causes

Chance, or common, causes are small random changes in the process that cannot be avoided. When this type of variation is suspected, production process continues as usual.
Assignable Causes

Assignable causes are large variations. When this type of variation is suspected, production process is stopped and a reason for variation is sought.

(a) Mean

Assignable Causes

Assignable causes are large variations. When this type of variation is suspected, production process is stopped and a reason for variation is sought.

(b) Spread
**Assignable Causes**

Assignable causes are large variations. When this type of variation is suspected, production process is stopped and a reason for variation is sought.

---

**The Normal Distribution**

\[ \sigma = \text{Standard deviation} \]

-3\(\sigma\)  -2\(\sigma\)  -1\(\sigma\)  +1\(\sigma\)  +2\(\sigma\)  +3\(\sigma\)

- 68.26% 95.44% 99.74%
Control Charts

Assignable causes likely

UCL
Nominal
LCL

Samples

1 2 3

Control Chart Examples

Variations
UCL
Nominal
LCL
Sample number
Control Limits and Errors

Type I error: Probability of searching for a cause when none exists

(a) Three-sigma limits

UCL

Process average

LCL

(b) Two-sigma limits

UCL

Process average

LCL
_type II error:
Probability of concluding
that nothing has changed

UCL
Shift in process average
Process average
LCL

(a) Three-sigma limits

Type II error:
Probability of concluding
that nothing has changed

UCL
Shift in process average
Process average
LCL

(b) Two-sigma limits
Control Charts For Variables

- Mean chart (\( \bar{X} \) Chart)
  - Measures central tendency of a sample
- Range chart (\( R \)-Chart)
  - Measures amount of dispersion in a sample
- Each chart measures the process differently. Both the process average and process variability must be in control for the process to be in control.

Constructing a Control Chart for Variables

1. Define the problem
2. Select the quality characteristics to be measured
3. Choose a rational subgroup size to be sampled
4. Collect the data
5. Determine the trial centerline for the \( \bar{X} \) chart
6. Determine the trial control limits for the \( \bar{X} \) chart
7. Determine the trial control limits for the \( R \) chart
8. Examine the process: control chart interpretation
9. Revise the charts
10. Achieve the purpose
Example: Control Charts for Variables

<table>
<thead>
<tr>
<th>Sample</th>
<th>Slip Ring Diameter (cm)</th>
<th>X</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.02 5.01 4.94 4.99 4.96</td>
<td>4.98</td>
<td>0.08</td>
</tr>
<tr>
<td>2</td>
<td>5.01 5.03 5.07 4.95 4.96</td>
<td>5.00</td>
<td>0.12</td>
</tr>
<tr>
<td>3</td>
<td>4.99 5.00 4.93 4.92 4.99</td>
<td>4.97</td>
<td>0.08</td>
</tr>
<tr>
<td>4</td>
<td>5.03 4.91 5.01 4.98 4.89</td>
<td>4.96</td>
<td>0.14</td>
</tr>
<tr>
<td>5</td>
<td>4.95 4.92 5.03 5.05 5.01</td>
<td>4.99</td>
<td>0.13</td>
</tr>
<tr>
<td>6</td>
<td>4.97 5.06 5.06 4.96 5.03</td>
<td>5.01</td>
<td>0.10</td>
</tr>
<tr>
<td>7</td>
<td>5.05 5.01 5.10 4.96 4.99</td>
<td>5.02</td>
<td>0.14</td>
</tr>
<tr>
<td>8</td>
<td>5.09 5.10 5.00 4.99 5.08</td>
<td>5.05</td>
<td>0.11</td>
</tr>
<tr>
<td>9</td>
<td>5.14 5.10 4.99 5.08 5.09</td>
<td>5.08</td>
<td>0.15</td>
</tr>
<tr>
<td>10</td>
<td>5.01 4.98 5.08 5.07 4.99</td>
<td>5.03</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Normal Distribution Review

- If the diameters are normally distributed with a mean of 5.01 cm and a standard deviation of 0.05 cm, find the probability that the sample means are smaller than 4.98 cm or bigger than 5.02 cm.
Normal Distribution Review

• If the diameters are normally distributed with a mean of 5.01 cm and a standard deviation of 0.05 cm, find the 97% confidence interval estimator of the mean (a lower value and an upper value of the sample means such that 97% sample means are between the lower and upper values).

Normal Distribution Review

• Define the 3-sigma limits for sample means as follows:

  \[ \text{Upper Limit} = \mu + \frac{3\sigma}{\sqrt{n}} = 5.01 + \frac{3(0.05)}{\sqrt{5}} = 5.077 \]

  \[ \text{Lower Limit} = \mu - \frac{3\sigma}{\sqrt{n}} = 5.01 - \frac{3(0.05)}{\sqrt{5}} = 4.943 \]

• What is the probability that the sample means will lie outside 3-sigma limits?
Normal Distribution Review

• Note that the 3-sigma limits for sample means are different from natural tolerances which are at $\mu \pm 3\sigma$

Determine the Trial Centerline for the $\bar{X}$ Chart

$$\bar{X} = \frac{\sum_{i=1}^{m} \bar{X}_i}{m} = \frac{50.09}{10} = 5.01$$

where $m =$ number of subgroups $= 10$
Determine the Trial Control Limits for the $\overline{X}$ Chart

$$\overline{R} = \frac{\sum_{i=1}^{m} R_i}{m} = \frac{1.15}{10} = 0.115$$

$$\text{UCL}_{\overline{X}} = \overline{X} + A_2 \overline{R} = (5.01) + 0.58(0.115) = 5.077$$

$$\text{LCL}_{\overline{X}} = \overline{X} - A_2 \overline{R} = (5.01) - 0.58(0.115) = 4.943$$

See p. 27 or Text Appendix 2 for the value of $A_2$

Note: The control limits are only preliminary with 10 samples. It is desirable to have at least 25 samples.

Determine the Trial Control Limits for the $R$ Chart

$$\text{UCL}_R = D_4 \overline{R} = (2.11)(0.115) = 2.43$$

$$\text{LCL}_R = D_3 \overline{R} = (0)(0.115) = 0$$

See p. 27 or Text Appendix 2 for the values of $D_3, D_4$
3-Sigma Control Chart Factors

<table>
<thead>
<tr>
<th>Sample size</th>
<th>$\bar{X}$-chart</th>
<th>R-chart</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td>$A_2$</td>
<td>$D_3$</td>
</tr>
<tr>
<td>2</td>
<td>1.88</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>1.02</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0.73</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>0.58</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>0.48</td>
<td>0</td>
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<tr>
<td>7</td>
<td>0.42</td>
<td>0.08</td>
</tr>
<tr>
<td>8</td>
<td>0.37</td>
<td>0.14</td>
</tr>
</tbody>
</table>

Examine the Process
Control-Chart Interpretation

- Decide if the variation is random (chance causes) or unusual (assignable causes).
- A process is considered to be in a state of control, or under control, when the performance of the process falls within the statistically calculated control limits and exhibits only chance, or common, causes.
Examine the Process
Control-Chart Interpretation

• A control chart exhibits a state of control when:
  1. Two-thirds of the points are near the center value.
  2. A few of the points are close to the center value.
  3. The points float back and forth across the centerline.
  4. The points are balanced on both sides of the centerline.
  5. There no points beyond the centerline.
  6. There are no patterns or trends on the chart.
     – Upward/downward, oscillating trend
     – Change, jump, or shift in level
     – Runs
     – Recurring cycles

Revise the Charts

A. Interpret the original charts
B. Isolate the cause
C. Take corrective action
D. Revise the chart: remove any points from the calculations that have been corrected. Revise the control charts with the remaining points
Revise the Charts

\[
\bar{X}_{\text{new}} = \frac{\sum \bar{X} - \bar{X}_d}{m - m_d}
\]
\[
\bar{R}_{\text{new}} = \frac{\sum R - R_d}{m - m_d}
\]

Where
\( \bar{X}_d \) = discarded subgroup averages
\( m_d \) = number of discarded subgroups
\( R_d \) = discarded subgroup ranges

The formula for the revised limits are:

\[
\bar{X}_0 = \bar{X}_{\text{new}}, R_0 = \bar{R}_{\text{new}}
\]
\[
\sigma_0 = \frac{R_0}{d_2}
\]
\[
\text{UCL}_\bar{X} = \bar{X}_0 + A\sigma_0
\]
\[
\text{LCL}_\bar{X} = \bar{X}_0 - A\sigma_0
\]
\[
\text{UCL}_R = D_2\sigma_0
\]
\[
\text{LCL}_R = D_1\sigma_0
\]

where, A, D_1, and D_2 are obtained from Appendix 2.
CHAPTER 5: CHI-SQUARE TEST

- Control chart is constructed using periodic samples from a process
- It is assumed that the subgroup means are normally distributed
- Chi-Square test can be used to verify if the above assumption
Chi-Square Test

• The Chi-Square statistic is calculated as follows:

\[ \chi^2 = \sum_{k} \frac{(f_0 - f_e)^2}{f_e} \]

• Where,
  \( k \) = number of classes or intervals
  \( f_0 \) = observed frequency for each class or interval
  \( f_e \) = expected frequency for each class or interval
  \( \sum_{k} \) = sum over all classes or intervals

• If \( \chi^2 = 0 \), then the observed and theoretical distributions match exactly.
• The larger the value of \( \chi^2 \), the greater the discrepancy between the observed and expected frequencies.
• The \( \chi^2 \) statistic is computed and compared with the tabulated, critical values recorded in a \( \chi^2 \) table. The critical values of \( \chi^2 \) are tabulated by degrees of freedom, \( \nu \) vs. the level of significance, \( \alpha \)
Chi-Square Test

• The null hypothesis, $H_0$, is that there is no significant difference between the observed and the specified theoretical distribution.

• If the computed $\chi^2$ test statistic is greater than the tabulated critical $\chi^2$ value, then the $H_0$ is rejected and it is concluded that there is enough statistical evidence to infer that the observed distribution is significantly different from the specified theoretical distribution.

• If the computed $\chi^2$ test statistic is not greater than the tabulated critical $\chi^2$ value, then the $H_0$ is not rejected and it is concluded that there is not enough statistical evidence to infer that the observed distribution is significantly different from the specified theoretical distribution.
Chi-Square Test

- The degrees of freedom, ν is obtained as follows
  \[ \nu = k - 1 - p \]

- Where,
  \( k \) = number of classes or intervals
  \( p \) = the number of population parameters estimated from the sample. For example, if both the mean and standard of population data are unknown and are estimated using the sample data, then \( p = 2 \)

- A note: When using the Chi-Square test, there must be a frequency or count of at least 5 in each class.

Example: Chi-Square Test

- 25 subgroups are collected each of size 5. For each subgroup, an average is computed and the averages are as follows:
  
  | Subgroup | Average  
  |----------|---------
  | 104.98722 | 99.716159 | 92.127891 | 93.79888 |
  | 97.004707 | 102.4385 | 99.61934 | 101.8301 |
  | 99.54862 | 95.82537 | 95.85889 | 100.2662 |
  | 92.82253 | 100.5916 | 99.67996 | 99.66757 |
  | 100.5447 | 105.8182 | 95.63521 | 97.52268 |
  | 100.2008 | 104.3002 | 102.5233 | 103.5716 |
  | 112.0867 |

- Verify if the subgroup data are normally distributed. Consider \( \alpha = 0.05 \)
Example: Chi-Square Test

• Step 1: Estimate the population parameters:

\[
\bar{X} = \frac{\sum_{i=1}^{n} \bar{x}_i}{n} = \frac{\sum_{i=1}^{25} \bar{x}_i}{25} = 99.92
\]

\[
s = \sqrt{\frac{\sum_{i=1}^{n} \left( X - \bar{x}_i \right)^2}{n-1}} = \sqrt{\frac{\sum_{i=1}^{25} \left( X - \bar{x}_i \right)^2}{25-1}} = 4.44
\]

Example: Chi-Square Test

• Step 2: Set up the null and alternate hypotheses:

• Null hypotheses, H₀: The average measurements of subgroups with size 5 are normally distributed with mean = 99.92 and standard deviation = 4.44

• Alternate hypotheses, Hₐ: The average measurements of subgroups with size 5 are not normally distributed with mean = 99.92 and standard deviation = 4.44
Example: Chi-Square Test

- Step 3: Consider the following classes (left inclusive) and for each class compute the observed frequency:

<table>
<thead>
<tr>
<th>Class Interval</th>
<th>Observed Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 - 97</td>
<td>6</td>
</tr>
<tr>
<td>97 - 100</td>
<td>7</td>
</tr>
<tr>
<td>100 - 103</td>
<td>7</td>
</tr>
<tr>
<td>103 - (\infty)</td>
<td>5</td>
</tr>
</tbody>
</table>
**Example: Chi-Square Test**

- Step 4: Compute the expected frequency in each class

<table>
<thead>
<tr>
<th>Class Interval</th>
<th>Expected Frequency $f_e$</th>
<th>$P(x \leq Z)$</th>
<th>$Z = \left(\frac{x - \mu}{\sigma}\right)$</th>
<th>$p_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 - 97</td>
<td>0.657</td>
<td>0.2546</td>
<td>-0.657</td>
<td>0.2546</td>
</tr>
<tr>
<td>97 - 100</td>
<td>0.018</td>
<td>0.5080</td>
<td>0.018</td>
<td>0.2534</td>
</tr>
<tr>
<td>100 - 103</td>
<td>0.693</td>
<td>0.7549</td>
<td>0.693</td>
<td>0.2469</td>
</tr>
<tr>
<td>103 - $\infty$</td>
<td>$\infty$</td>
<td>1.0000</td>
<td>$\infty$</td>
<td>0.2451</td>
</tr>
</tbody>
</table>

- The Z-values are computed at the upper limit of the class
Example: Chi-Square Test

• Sample computation for Step 4:
• Class interval 0-97

\[ Z = (97 - 99.92)/4.44 = -0.657 = -0.66 \]

For \( z = -0.66 \) cumulative area on the left,
\[ P(x \leq Z) = 0.2546 \text{ (See Appendix 1)} \]
Hence, \( p_i = P(0 \leq \bar{x}_i \leq 97) = 0.2546 \)
\[ f_i = n \times p_i = 25 \times 0.2546 = 6.365 \]

Example: Chi-Square Test

• Sample computation for Step 4:
• Class interval 97-100

\[ Z = (100 - 99.92)/4.44 = 0.018 = 0.02 \]

For \( z = 0.02 \) cumulative area on the left,
\[ P(x \leq Z) = 0.5080 \text{ (See Appendix 1)} \]
Hence, \( p_2 = P(97 \leq \bar{x}_i \leq 100) = 0.5080 - 0.2546 = 0.2534 \)
\[ f_i = n \times p_2 = 25 \times 0.2534 = 6.335 \]
Example: Chi-Square Test

• Sample computation for Step 4:
  • Class interval 100 - 103

\[ Z = \frac{(103 - 99.92)}{4.44} = 0.693 \approx 0.69 \]

For \( z = 0.69 \) cumulative area on the left,
\[ P(x \leq Z) = 0.7549 \text{ (See Appendix 1)} \]

Hence, \( p_3 = P(100 \leq \bar{x} \leq 103) = 0.7549 - 0.5080 = 0.2469 \)
\[ f_e = n \times p_3 = 25 \times 0.2469 = 6.1725 \]

Example: Chi-Square Test

• Step 5: Compute the Chi-Square test statistic:

\[ \chi^2 = \sum \frac{(f_0 - f_e)^2}{f_e} \]
**Example: Chi-Square Test**

- Step 5: Compute the Chi-Square test statistic:

\[
\chi^2 = \sum_{k=1}^{k} \frac{(f_0 - f_e)^2}{f_e}
\]

\[
= \frac{(6 - 6.365)^2}{6.365} + \frac{(7 - 6.335)^2}{6.335} + \frac{(7 - 6.1725)^2}{6.1725} + \frac{(5 - 6.1275)^2}{6.1275}
\]

\[
= 0.0209 + 0.0698 + 0.1109 + 0.2075
\]

\[
= 0.4091
\]

- Step 6: Compute the degrees of freedom, \( \nu \) and the critical \( \chi^2 \) value

  - There are 4 classes, so \( k = 4 \)
  - Two population parameters, mean and standard deviation are estimated, so \( p = 2 \)
  - Degrees of freedom,

\[
\nu = k - 1 - p =
\]

  - From \( \chi^2 \) Table, the critical

\[
\chi^2 =
\]
Example: Chi-Square Test

• Step 6: Compute the degrees of freedom, $\nu$ and the critical $\chi^2$ value
  – There are 4 classes, so $k = 4$
  – Two population parameters, mean and standard deviation are estimated, so $p = 2$
  – Degrees of freedom,
    
    $\nu = k - 1 - p = 4 - 1 - 2 = 1$
  
  – From $\chi^2$ Table, the critical
    
    $\chi^2 = 3.84$

Example: Chi-Square Test

• Step 7:
  – Conclusion:
    
    $\chi^2_{test} = 0.4091 < \chi^2_{critical} = 3.84$
    
    Do not reject the $H_0$
  
  – Interpretation:
    
    There is not enough statistical evidence to infer at the 5% level of significance that the average measurements of subgroups with size 5 are not normally distributed with $\mu = 99.92$ and $\sigma_x = 4.44$
Reading and Exercises

• Chapter 5:
  – Reading handout pp. 50-53

Economic Design of $\bar{X}$ Control Chart

• Design involves determination of:
  – Interval between samples (determined from considerations other than cost).
  – Size of the sample ($n =$ ?)
  – Upper and lower control limits ($k =$ ?)
  
  \[ LCL = \mu - \frac{k\sigma}{\sqrt{n}} \quad \text{UCL} = \mu + \frac{k\sigma}{\sqrt{n}} \]

• Determine $n$ and $k$ to minimize total costs related to quality control
Relevant Costs for $\bar{X}$ Control Chart Design

• Sampling cost
  – Personnel cost, equipment cost, cost of item etc
  – Assume a cost of $a_1$ per item sampled. Sampling cost = $a_1 n$

Relevant Costs for $\bar{X}$ Control Chart Design

• Search cost (when an out-of-control condition is signaled, an assignable cause is sought)
  – Cost of shutting down the facility, personnel cost for the search, and cost of fixing the problem, if any
  – Assume a cost of $a_2$ each time a search is required
• Question: Does this cost increase or decrease with the increase of $k$?
Relevant Costs for $\bar{X}$ Control Chart Design

- Cost of operating out of control
  - Scrap cost or repair cost
  - A defective item may become a part of a larger subassembly, which may need to be disassembled or scrapped at some cost
  - Costs of warranty claims, liability suits, and overall customer dissatisfaction
  - Assume a cost of $a_3$ each period that the process is operated in an out-of-control condition
- Question: Does this cost increase or decrease with the increase of $k$?

Procedure for Finding $n$ and $k$ for Economic Design of $\bar{X}$ Control Chart

Inputs
- $a_1$  cost of sampling each unit
- $a_2$  expected cost of each search
- $a_3$  per period cost of operating in an out-of-control state
- $\pi$  probability that the process shifts from an in-control state to an out-of-control state in one period
- $\delta$  average number of standard deviations by which the mean shifts whenever the process is out-of-control. In other words, the mean shifts from $\mu$ to $\mu \pm \delta \sigma$ whenever the process is out-of-control.
**Procedure for Finding $n$ and $k$ for Economic Design of $\bar{X}$ Control Chart**

**The Key Step**
- A trial and error procedure may be followed
- The minimum cost pair of $n$ and $k$ is sought
- For a given pair of $n$ and $k$ the average per period cost is
  \[
  a_c n + a_a (1 - \beta) \left[ \pi + \alpha (1 - \pi) \right] + a_v \pi 
  \]
  \[
  1 - \beta (1 - \pi)
  \]

where

\[\Phi(z)\] is the cumulative standard normal distribution function

Approximately, \[\Phi(z)\] may also be obtained from Table A1/A4 or Excel function NORMSDIST

\[\alpha\] is the type I error \quad \Rightarrow \quad 2\Phi(-k)

\[\beta\] is the type two error \quad \Rightarrow \quad \Phi(k - \delta \sqrt{n}) - \Phi(-k - \delta \sqrt{n}) and

\[\Phi(z)\] is the cumulative standard normal distribution function

Approximately,

\[\Phi(z) = 0.500232 - 0.212159z^{2.08388} + 0.5170198z^{1.068529} + 0.041111z^{2.82894}\]
A Trial and Error Procedure using Excel Solver

• Consider some trial values of $n$
• For each trial value of $n$, the best value of $k$ may be obtained by using Excel Solver:
  – Write the formulae for $\alpha$, $\beta$ and cost
  – Set up Excel Solver to minimize cost by changing $k$ and assuming $k$ non-negative

Procedure for Finding $n$ and $k$ for Economic Design of $\bar{X}$ Control Chart

Notes

• Expected number of periods that the system remains in control (there may be several false alarms during this period) following an adjustment
  \[
  E(T) = \frac{1 - \pi}{\pi}
  \]
• Expected number of periods that the system remains out of control until a detection is made
  \[
  E(S) = \frac{1}{1 - \beta}
  \]
• Expected number of periods in a cycle, $E(C) = E(T) + E(S)$
Notes

• Expected cost of sampling per cycle = \( a_i n E(C) \)
• Expected cost of searching per cycle = \( a_s [1 + \alpha E(T)] \)
• Expected cost of operating in an out-of-control state = \( a_o E(S) \) per cycle
• To get the expected costs per period divide expected costs per cycle by \( E(C) \)

Problem 10-23 (Handout): A quality control engineer is considering the optimal design of an \( \bar{X} \) chart. Based on his experience with the production process, there is a probability of 0.03 that the process shifts from an in-control to an out-of-control state in any period. When the process shifts out of control, it can be attributed to a single assignable cause; the magnitude of the shift is \( 2\sigma \). Samples of \( n \) items are made hourly, and each sampling costs $0.50 per unit. The cost of searching for the assignable cause is $25 and the cost of operating the process in an out-of-control state $300 per hour.

a. Determine the hourly cost of the system when \( n=6 \) and \( k=2.5 \).

b. Estimate the optimal value of \( k \) for the case \( n=6 \).

c. determine the optimal pair of \( n \) and \( k \).
We have
\[ \pi = 0.03, \delta = 2 \]
\[ a_1 = 0.50/\text{unit}, \quad a_2 = 25/\text{search}, \quad a_3 = 300/\text{hour} \]
Given, \( n = 6, \quad k = 2.5 \)
The Type I error, \( \alpha = 2\Phi(-k) = 2\Phi(-2.5) = 2(1 - \Phi(2.5)) \)
\[ = 2(1 - 0.9938) \text{ from Table A - 4} \]
\[ = 0.0124 \]
For \( \delta = 2 \), the Type II error, \( \beta = \Phi(k - \delta\sqrt{n}) - \Phi(-k - \delta\sqrt{n}) \)
\[ = \Phi(2.5 - 2\sqrt{6}) - \Phi(-2.5 - 2\sqrt{6}) \]
\[ = \Phi(-2.40) - \Phi(-7.40) = \Phi(-2.40) - 0 = \Phi(-2.40) \]
\[ = 1 - \Phi(2.40) \]
\[ = 1 - 0.9918 \text{ from Table A - 4} \]
\[ = 0.0082 \]

\[
E(T) = \frac{1 - \pi}{\pi} = \frac{1 - 0.03}{0.03} = 32.33
\]
\[
E(S) = \frac{1}{1 - \beta} = \frac{1}{1 - 0.0082} = 1.0083
\]
\[
E(C) = E(T) + E(S) = 32.33 + 1.0083 = 33.34
\]
Cost of sampling per cycle
\[ = a_1nE(C) = 0.50(6)(33.34) = 100.02 \]
Cost of searching per cycle
\[ = a_2[1 + \alpha E(T)] = 25[1 + 0.0124(32.33)] = 35.02 \]
Cost of operating in out - of - control condition per cycle
\[ = a_3E(S) = 300(1.0083) = 302.49 \]
Cost per period
\[ = \frac{100.02 + 35.02 + 302.49}{E(C)} = \frac{437.53}{33.34} = 13.12 \]
## Economic design of X-bar control chart

### Inputs
- **Sampling cost**, $a_1$ per item
- **Search cost**, $a_2$ per search
- **Cost of operating out of control**, $a_3$ per period
- **Prob(out-of-control in one period)**, $\pi$
- **Av. shift of mean in out-of-control**, $\delta$ per sigma

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<th>$k$</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>Cost</th>
</tr>
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<td>13.032513</td>
</tr>
</tbody>
</table>

**Click on the above spreadsheet to edit it**
**Reading**


---

**X̄ and s Chart**

- The $\bar{X}$ chart shows the center of the measurements and the $R$ chart the spread of the data.
- An alternative combination is the $\bar{X}$ and $s$ chart. The $\bar{X}$ chart shows the central tendency and the $s$ chart the dispersion of the data.
\( \bar{X} \) and \( s \) Chart

- Why \( s \) chart instead of \( R \) chart?
  - Range is computed with only two values, the maximum and the minimum. However, \( s \) is computed using all the measurements corresponding to a sample.
  - So, an \( R \) chart is easier to compute, but \( s \) is a better estimator of standard deviation specially for large subgroups.

\[ \bar{X} \text{ and } s \text{ Chart} \]

- Previously, the value of \( \sigma \) has been estimated as: \( \frac{\bar{R}}{d_2} \)
- The value of \( \sigma \) may also be estimated as: \( \frac{s}{c_4} \)

where, \( s \) is the sample standard deviation and \( c_4 \) is as obtained from Appendix 2

- Control limits may be different with different estimators of \( \sigma \) (i.e., \( \bar{R} \) and \( s \) )
\( \bar{X} \) and \( s \) Chart

- The control limits of \( \bar{X} \) chart are
  \[
  \bar{X} \pm 3 \frac{\sigma}{\sqrt{n}} = \bar{X} \pm 3 \frac{s}{c_4 \sqrt{n}}
  \]

- The above limits can also be written as \( \bar{X} \pm A \sigma \) or \( \bar{X} \pm A_1 \bar{s} \)
  Where
  \[
  A = \frac{3}{\sqrt{n}}, \quad A_1 = \frac{3}{c_4 \sqrt{n}}
  \]
  so, \( A = A_1 c_4 \) (Appendix 2 gives the values of \( A \) and \( A_1 \), check)

\( \bar{X} \) and \( s \) Chart: Trial Control Limits

- The trial control limits for \( \bar{X} \) and \( s \) charts are:
  \[
  \begin{align*}
  \text{UCL}_{\bar{X}} &= \bar{X} + A_1 \bar{s} \\
  \text{LCL}_{\bar{X}} &= \bar{X} - A_1 \bar{s} \\
  \text{UCL}_s &= B_4 \bar{s} \\
  \text{LCL}_s &= B_3 \bar{s}
  \end{align*}
  \]
  Where, the values of \( A_1, B_3 \) and \( B_4 \) are as obtained from Appendix 2
  \[
  \bar{X} = \frac{\sum_{i=1}^{m} X_i}{m}, \quad \bar{s} = \frac{\sum_{i=1}^{m} s_i}{m}
  \]
  \( m = \) the number of subgroups
**$\bar{X}$ and $s$ Chart: Trial Control Limits**

- For large samples:

$$c_4 \approx 1, A_3 \approx \frac{3}{\sqrt{n}}$$

$$B_3 \approx 1 - \frac{3}{\sqrt{2n}}, B_4 \approx 1 + \frac{3}{\sqrt{2n}}$$

**$\bar{X}$ and $s$ Chart: Revised Control Limits**

- The control limits are revised using the following formula:

$$\bar{X}_0 = \bar{X}_{new} = \frac{\sum \bar{X} - \bar{X}_d}{m-m_d}$$

$$s_0 = s_{new} = \frac{\sum s - s_d}{m-m_d}$$

Where

- $\bar{X}_d$ = discarded subgroup averages
- $m_d$ = number of discarded subgroups
- $R_d$ = discarded subgroup ranges

Continued...
The $\bar{X}$ and $s$ Chart: Revised Control Limits

and

\[
\sigma_0 = \frac{s_0}{c_4} \\
UCL_{\bar{X}} = \bar{X}_0 + A\sigma_0 \\
LCL_{\bar{X}} = \bar{X}_0 - A\sigma_0 \\
UCL_s = B_6\sigma_0 \\
LCL_s = B_4\sigma_0
\]

where, $A$, $B_5$, and $B_6$ are obtained from Appendix 2.

---

**Example 1**

- A total of 25 subgroups are collected, each with size 4. The $\bar{X}$ and $s$ values are as follows:

  $\bar{X}$
  

  \[\sum_{i=1}^{25} \bar{X}_i = 160.25\]

  $s$
  
  0.034, 0.045, 0.028, 0.045, 0.042, 0.041, 0.024, 0.034, 0.018, 0.045, 0.014, 0.020, 0.051, 0.032, 0.036, 0.042, 0.056, 0.125, 0.025, 0.054, 0.036, 0.029, 0.024, 0.036, 0.029

  \[\sum_{i=1}^{25} s_i = 0.965\]

Compute the trial control limits of the $\bar{X}$ and $s$ chart.
Example 2

• Compute the revised control limits of the $\bar{X}$ and $s$ chart obtained in Example 1.

Reading and Exercises

• Chapter 5
  – Reading pp. 236-242, Exercises 15, 16 (2nd ed.)
  – Reading pp. 240-247, Exercises 15, 16 (3rd ed.)
CHAPTER 6: PROCESS CAPABILITY

Outline

• Process capability: individual values and specification limits
• Estimation of standard deviation
• The 6σ spread versus specification limits
• The process capability indices

Process Capability
Individual Values and Specification Limits

• Specifications are set by the customer. These are the “wishes.”
• Control limits are obtained by applying statistical rules on the data generated by the process. These are the “reality.”
• Process capability refers to the ability of a process to meet the specifications set by the customer or designer
Process Capability
Individual Values and Specification Limits

• Process capability is based on the performance of individual products or services (not the subgroup averages) against specifications set by the customer or designer (not the statistically computed control limits)

• Process capability is different from the state of process control which is determined by control charts and based on the average performances of subgroups against statistically computed control limits.

• The difference between process capability and the state of process control
  – When measurement of an individual item does not meet specification, the item is called defective
  – When subgroup averages are compared with control limits and the comparison shows some unpredictable amount of variation, an out-of-control state is assumed
  – A process in statistical control does not necessarily follow specifications. A capable process is not necessarily in control. A process may be out of control and within specification or under control and out of specification.
Estimation of Standard Deviation

- The population standard deviation, $\hat{\sigma}$, can be estimated from the sample standard deviation, $s$ or range, $R$ as follows:
  
  $\hat{\sigma} = \frac{s}{c_4}$ or $\hat{\sigma} = \frac{R}{d_2}$

  (See Appendix 2 for the values of $c_4$ and $d_2$)

- Note: The population standard deviation is denoted by $\sigma$. However, the estimate of the population standard deviation is denoted by $\hat{\sigma}$
Estimation of Standard Deviation

• While the process capability is based on the population standard deviation, \( \sigma \), the state of process control is based on \( \sigma_x \), the standard deviation of the subgroup averages. Of course, the two quantities are related. For a subgroup sample size, \( n \), we get

\[
\sigma_x = \frac{\sigma}{\sqrt{n}}
\]

The 6\( \sigma \) Spread Versus Specification Limits

• Suppose that process mean = \((USL+LSL)/2\)
• Case 1: \(6\sigma < USL - LSL\)
  – Ideal condition, the process will remain within the specifications even after some shift.
• Case 2: \(6\sigma = USL - LSL\)
  – As long as the process remains in control and centered (so the process does not shift), the parts produced will be within specification.
• Case 3: \(6\sigma > USL - LSL\)
  – Undesirable situation, the process is incapable of meeting the specifications. Management intervention is needed in order to decrease the variation.
The 6σ Spread Versus Specification Limits

• Calculating 6σ
  1. Take at least 20 subgroups of sample size 4 for a total of 80 measurements
  2. Calculate the sample standard deviation, $s_i$, for each subgroup
  3. Calculate the average sample standard deviation, $\bar{s}$

$$\bar{s} = \frac{\sum_{i=1}^{m} s_i}{m}, \quad 6\sigma = 6 \frac{\bar{s}}{c_4}$$

Where $m = \text{number of subgroups}$ and $c_4$ is obtained from Appendix 2

The 6σ Spread Versus Specification Limits

• Alternate method of calculating 6σ
  1. Take the past 10 subgroups of sample size 4 or more
  2. Calculate the range, $R_i$, for each subgroup
  3. Calculate the average sample standard deviation, $\bar{s}$

$$\bar{R} = \frac{\sum_{i=1}^{m} R_i}{m}, \quad 6\sigma = 6 \frac{\bar{R}}{d_2}$$

Where $m = \text{number of subgroups}$ and $d_2$ is obtained from Appendix 2
The $6\sigma$ Spread Versus Specification Limits

- Capability index \[ C_p = \frac{USL - LSL}{6\sigma} \]

- If the capability index is larger than 1.00, a Case 1 situation exists. This is desirable. The greater this value, the better. The process will remain capable even after a slight shift of the process mean.

- If the capability index is equal to 1.00, a Case 2 situation exists. This is not the best. However, if the process is in control and the mean is centered, the process is capable.

- The capability ratio: \[ C_r = \frac{6\sigma}{USL - LSL} \]

- The capability ratio is the inverse of the capability index and interpreted similarly. A capability ratio of less than 1 is the most desirable situation.

- The $C_p$ or $C_r$ values do not reflect process centering.
The 6σ Spread Versus Specification Limits

- \( C_{pk} \)
  \[
  Z(\text{LSL}) = \frac{\bar{X} - LSL}{\sigma}, Z(\text{USL}) = \frac{USL - \bar{X}}{\sigma}
  \]
  \[
  C_{pk} = \frac{Z(\min)}{3}
  \]

- When \( C_{pk} \) has a value at least 1.00, the process is capable and follows specifications.
- When \( C_{pk} \) has a value less than 1.00, the process is not capable and does not follow specifications.

The 6σ Spread Versus Specification Limits

- When the process is centered, \( C_p = C_{pk} \).
- \( C_{pk} \) is always less than or equal to \( C_p \).
- If \( C_p > 1 \) and
  - If the process is in control and centered, then the process is capable.
  - If the process is not centered, the process may or may not be capable.
- If \( C_p < 1 \), the process is not capable.
Example 3

- A hospital is using $\bar{X}$ and $R$ charts to record the time it takes to process patient account information. A sample of 5 applications is taken each day. The first four weeks’ (20 days’) data give the following values:

$$\bar{X} = 16\text{ min and } R = 7\text{ min}$$

If the upper and lower specifications are 21 minutes and 13 minutes, respectively, calculate

$$6\sigma, \ C_p, \text{ and } C_{pk}$$

and interpret the indices.

Example 4

- A certain manufacturing process has been operating in control at a mean $\mu$ of 65.00 mm with upper and lower control limits on the chart of 65.225 and 64.775 respectively. The process standard deviation is known to be 0.15 mm, and specifications on the dimensions are 65.00±0.50 mm.

(a) What is the probability of not detecting a shift in the mean to 64.75 mm on the first subgroup sampled after the shift occurs. The subgroup size is four.

(b) What proportion of nonconforming product results from the shift described in part (a)? Assume a normal distribution of this dimension.

(c) Calculate the process capability indices $C_p$ and $C_{pk}$ for this process, and comment on their meaning relative to parts (a) and (b).
Reading and Exercises

• Chapter 6
  – Reading pp. 280-303, Exercises 3, 4, 11, 13 (2nd ed.)
  – Reading pp. 286-309, Exercises 3, 4, 9, 13 (3rd ed.)

CHAPTER 7: OTHER VARIABLE CONTROL CHARTS

Outline

• Individual and moving-range charts
• Moving-average and moving-range charts
• A chart plotting all individual values
• Median and range charts
• Run charts
• A chart for variable subgroup size
• Pre-control charts
• Short-run charts
Individual and Moving-Range Charts

- This chart is useful when the number of products produced is too small to form traditional $\bar{X}$ and $R$ charts and data collection occurs either once a day, or on a week-to-week or month-to-month basis.
- The individual $X_i$ measurements are taken and plotted on the individual chart.
- Two consecutive individual data-point values are compared and the absolute value of their difference is recorded on the moving-range chart. The moving-range is usually placed on the $R$ chart between the space designated for a value and its preceding value.

**Formula:**

Let $m =$ the number of $X_i$ values

$$\bar{X}_i = \frac{\sum X_i}{m}, \quad \bar{R} = \frac{\sum R_i}{m-1}$$

$$UCL_{X_i} = \bar{X}_i + 3 \frac{\sigma}{\sqrt{n}} = \bar{X}_i + 3 \frac{\bar{R}}{d_2 \sqrt{n}}$$

$$= \bar{X}_i + 3 \frac{\bar{R}}{1.128 \sqrt{1}} = \bar{X}_i + 2.66\bar{R}$$

$$LCL_{X_i} = \bar{X}_i - 2.66\bar{R}$$

$$UCL_R = D_4 \bar{R} = 3.27 \bar{R}$$

$$LCL_R = 0$$
Individual and Moving-Range Charts

- Interpretation of individual and moving-range charts is similar to that of $\bar{X}$ and $R$ charts.
- Once the process is considered in control, the process capability can be determined.
- Individual and moving-range charts are more reliable when the number of samples taken exceeds 80.

Text Problem 7.1: Create a chart for individuals with a moving range for the measurements given below. (Values are coded 21 for 0.0021 mm.) After determining the limits, plotting the values, and interpreting the chart, calculate $\sigma$ using $\overline{R}/d_2$. Is the process capable of meeting the specifications of 0.0025±0.0005 mm?

21 22 22 23 24 25 24 26 27 27 25
26 23 23 25 25 26 23 24 24 22 23 25

Check : $\sum X = 604, \sum R = 26$
Moving-Average and Moving-Range Charts

- $\bar{X}$ and $R$ charts track the performance of processes that have long production runs or repeated services.
- Sometimes, there may be insufficient number of sample measurements to create a traditional $\bar{X}$ and $R$ chart.
- For example, only one sample may be taken from a process.
- Rather than plotting each individual reading, it may be more appropriate to use moving average and moving range charts to combine $n$ number of individual values to create an average.
Moving-Average and Moving-Range Charts

- When a new individual reading is taken, the oldest value forming the previous average is discarded.
- The range and average are computed from the most recent $n$ observations.
- This is quite common in continuous process chemical industry, where only one reading is possible at a time.

Moving-Average and Moving-Range Charts

- **Use for continuous process chemical industry:** The moving average is particularly appropriate in continuous process chemical manufacture. The smoothing effect of the moving average often has an effect on the figures similar to the effect of the blending and mixing that take place in the remainder of the production process.
- **Use for seasonal products:** By combining individual values produced over time, moving averages smooth out short term variations and provide the trends in the data. For this reason, moving average charts are frequently used for seasonal products.
Moving-Average and Moving-Range Charts

- Interpretation:
  - a point outside control limits
    • interpretation is same as before - process is out of control
  - runs above or below the central line or control limits
    • interpretation is not the same as before - the successive points are not independent of one another

Text Problem 7.6 (7.4): Eighteen successive heats of a steel alloy are tested for $R_c$ hardness. The resulting data are shown below. Set up control limits for the moving-average and moving-range chart for a sample size of $n=3$.

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<tr>
<th>Heat</th>
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<th>Heat</th>
<th>Hardness</th>
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</thead>
<tbody>
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<td>0.809</td>
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</tr>
<tr>
<td>3</td>
<td>0.810</td>
<td>12</td>
<td>0.810</td>
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<td>4</td>
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<td>0.812</td>
</tr>
<tr>
<td>5</td>
<td>0.819</td>
<td>14</td>
<td>0.810</td>
</tr>
<tr>
<td>6</td>
<td>0.815</td>
<td>15</td>
<td>0.809</td>
</tr>
<tr>
<td>7</td>
<td>0.817</td>
<td>16</td>
<td>0.807</td>
</tr>
<tr>
<td>8</td>
<td>0.810</td>
<td>17</td>
<td>0.807</td>
</tr>
<tr>
<td>9</td>
<td>0.811</td>
<td>18</td>
<td>0.800</td>
</tr>
</tbody>
</table>
Example:

Eighteen successive heats of a steel alloy are tested for $R_C$ hardness. The resulting data are shown below. Set up control limits for the moving-average and moving-range chart for a sample size of $n = 3$.

<table>
<thead>
<tr>
<th>Heat</th>
<th>Hardness Average</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.806</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.814</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.810</td>
<td>0.008</td>
</tr>
<tr>
<td>4</td>
<td>0.820</td>
<td>0.010</td>
</tr>
<tr>
<td>5</td>
<td>0.819</td>
<td>0.010</td>
</tr>
<tr>
<td>6</td>
<td>0.815</td>
<td>0.005</td>
</tr>
<tr>
<td>7</td>
<td>0.817</td>
<td>0.004</td>
</tr>
<tr>
<td>8</td>
<td>0.810</td>
<td>0.007</td>
</tr>
<tr>
<td>9</td>
<td>0.811</td>
<td>0.007</td>
</tr>
<tr>
<td>10</td>
<td>0.809</td>
<td>0.010</td>
</tr>
<tr>
<td>11</td>
<td>0.808</td>
<td>0.009</td>
</tr>
<tr>
<td>12</td>
<td>0.809</td>
<td>0.002</td>
</tr>
<tr>
<td>13</td>
<td>0.812</td>
<td>0.004</td>
</tr>
<tr>
<td>14</td>
<td>0.810</td>
<td>0.002</td>
</tr>
<tr>
<td>15</td>
<td>0.809</td>
<td>0.003</td>
</tr>
<tr>
<td>16</td>
<td>0.807</td>
<td>0.002</td>
</tr>
<tr>
<td>17</td>
<td>0.807</td>
<td>0.003</td>
</tr>
<tr>
<td>18</td>
<td>0.800</td>
<td>0.007</td>
</tr>
</tbody>
</table>

$$
\overline{X} = \frac{\sum X}{m} = \frac{0.810 + 0.815 + 0.816 + \ldots + 0.805}{16} = 0.811
$$

$$
\overline{R} = \frac{\sum R}{m} = \frac{0.008 + 0.010 + 0.010 + \ldots + 0.007}{16} = 0.005
$$

Where $m = \text{number of } \overline{X_i} \text{ readings}$

$$
\text{UCL}_X = \overline{X} + A_2 \overline{R} = 0.811 + (1.02)(0.005) = 0.816
$$

$$
\text{LCL}_X = \overline{X} - A_2 \overline{R} = 0.811 - (1.02)(0.005) = 0.806
$$

$$
\text{UCL}_R = D_4 \overline{R} = (2.57)(0.005) = 0.013
$$

$$
\text{LCL}_R = D_3 \overline{R} = (0)(0.005) = 0
$$
A Chart Plotting All Individual Values

- Control charts that plot each individual value are useful when explaining the concept of variation.
- On an individual values chart, the individual values from the subgroup are represented by a small mark. The average of those values is represented by a circle.
- Usually, the chart is reserved for training people on interpreting the values of $R$ and $s$ charts.
Median and Range Charts

- When a median chart is used, the median of each subgroup is calculated and plotted.
- The centerline of the median chart is the average of the subgroup medians.
- The control limits of the median chart are determined using the formula shown in the next slide.
- The procedure of construction of the range chart is the same as before.
- The median chart is easy to compute. However, some sensitivity is lost.

Median and Range Charts

- Formula: Let $m =$ the number of subgroups.
  \[ \bar{X}_{Md} = \frac{\sum \text{Median}_i}{m}, \quad \bar{R}_{Md} = \frac{\sum R_i}{m} \]
  \[ UCL_{Md} = \bar{X}_{Md} + A_4 \bar{R} \]
  \[ LCL_{Md} = \bar{X}_{Md} - A_4 \bar{R} \]
  \[ UCL_{Md} = D_4 \bar{R}_{Md} \]
  \[ LCL_{Md} = D_3 \bar{R}_{Md} \]
  For an alternative set of formula, see pp.330-334 (not covered in class/exam)
Run Charts

• A run chart is a line graph that shows data points plotted in the order in which they occur.
• They are used to show trends and shifts in a process over time, variation over time, or to identify decline or improvement in a process over time.
• They can be used to examine both variables and attribute data.
• Time is displayed on the $x$ axis and the variable or attribute being investigated is recorded on the $y$ axis.
• There are no control limits in a run chart.
Run Charts

A run chart is constructed in five steps:
1. Determine the time increments.
2. Scale the y axis to reflect the values that the measurements or attributes data will take.
3. Collect the data.
4. Record the data on the chart.
5. Interpret the chart.
A Chart for Variable Subgroup Size

• Traditional variable control charts are constructed using a constant subgroup sample size.
• If the subgroup size varies, it is necessary to recalculate the control limits for every subgroup size. Each subgroup with a different sample size will have its own control limits plotted on the chart.
• If the subgroup size increases, the control limits will be closer to the centerline. The gap between two control limits will be narrower.

Precontrol Charts

• Precontrol charts do not use the process data to calculate the control limits. The control limits are calculated using specifications.
• Precontrol charts
  – are simple to set up and use.
  – can be used with either variable and attribute data.
  – useful during setup operations - can determine if the process setup is producing product within tolerances
  – can identify if the process center has shifted or the spread has increased
Precontrol Charts

- Precontrol charts cannot be used to study process capability.
- Precontrol charts may generate more false alarms or missed signals than the control charts.

1. Create the zones:
   - Red zone: Outside specification limits.
   - Yellow zone: Inside specification limits. One of the specification limits is closer than the center of the specification.
   - Green zone: Inside specification limits. The center of the specification is closer than both the specification limits.
Precontrol Charts

2. Take measurements and apply the setup rules:
   – Point in green zone: continue until five successive pieces
     are in the green zone
   – Point in yellow zone: check another piece
   – Two points in a row in the same yellow zone: reset the
     process
   – Two points in a row in opposite yellow zone: stop, reduce
     variation, reset the process
   – Point in the red zone: stop, make correction and reset
     the process

Precontrol Charts

3. Apply the precontrol sampling plan:
   – Once five successive points fall in the green zone,
     discontinue 100% inspection and start sampling.
   – Randomly select two pieces at interval:
     • A point in the red zone: stop, adjust process to
       remove variation, reset
     • Two points in the opposite yellow zone: stop, adjust
       process to remove variation
     • Two points in the same yellow zone: adjust process
       to remove variation
     • Otherwise: Continue
Text Problem 7.16 (7.13): NB Manufacturing has ordered a new machine. During today’s runoff the following data were gathered concerning the runout for the diameter of the shaft machine by this piece of equipment. A precontrol chart was used to set up the machines. Recreate the precontrol chart from the following data. The tolerance associated with this part is a maximum runout value of 0.002 (upper specification). The optimal value is 0.000 (no runout), the lower specification limit.

0.0021 0.0013 0.0018 0.0007 0.0002
0.0030 0.0024 0.0006 0.0002 0.0006
0.0004 0.0003 0.0010 0.0015 0.0011

Short-Run Charts

• Traditional variable control charts work with long, continuous production runs.
• Several methods have been developed for shorter production runs:
  – inspect the first and last pieces: gives no information on the pieces in between
  – 100% inspection: expensive and time consuming
  – separate charts for each part number: a large number of charts, insufficient number of observations to calculate the control limits
Short-Run Charts

- Short-run charts include multiple part numbers on the same chart.
- As the short-run charts display multiple part numbers, it is possible to visualize the variation in the process.
- The data are coded so that all the data, regardless of the part number, are scaled to a common denominator. So, there is a common set of control limits for all the parts numbers.
- The nominal $\bar{X}$ and $R$ charts use coded measurements based on nominal dimension.
- Coded measurement = Actual measurement - nominal value

Use the following steps to create a nominal $\bar{X}$ and $R$ chart:
1. Determine parts that will be monitored with the same chart.
   - parts made by the same operator, machine, methods, materials and measurement techniques
2. Determine the nominal specification for each part number.
3. Collect data - use the same subgroup sample size for all part numbers.
4. For each measurement, compute coded value = actual measurement - nominal value. Compute average measurement $\bar{X}$ for each subgroup.
Short-Run Charts

5. Plot the average measurements from step 4 on the chart.
6. Continue to calculate, code, and plot measurements for the entire run of the same part number.
7. When another part number is to be run, repeat steps 1 to 6.
8. When 20 subgroups have been plotted from any combination of parts, calculate control limits. Use the formula on the next slide.
9. Draw the centerline and control limits on the chart.
10. Interpret the chart.

Short-Run Charts

Step 8:
Nominal $\bar{X}$ chart

Centerline = $\frac{\sum \text{coded } \bar{X}}{m}$

$UCL_{\bar{X}} = \text{centerline} + A_2 \bar{R}$, $LCL_{\bar{X}} = \text{centerline} - A_2 \bar{R}$

Nominal $R$ chart

$\bar{R} = \frac{\sum R_i}{m}$

$UCL_R = D_4 \bar{R}$, $LCL_R = D_3 \bar{R}$
Text Problem 7.19 (7.16): A series of pinion gears for a van set recliner are fine-blanked on the same press. The nominal part diameters are small (50.8 mm), medium (60.2 mm), and large (70.0 mm). Create a short-run control chart for the data shown on the next slide:

<table>
<thead>
<tr>
<th>60.1</th>
<th>70.0</th>
<th>50.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>60.2</td>
<td>70.1</td>
<td>50.9</td>
</tr>
<tr>
<td>60.4</td>
<td>70.1</td>
<td>50.8</td>
</tr>
<tr>
<td>60.2</td>
<td>70.2</td>
<td>51.0</td>
</tr>
<tr>
<td>60.3</td>
<td>70.0</td>
<td>51.0</td>
</tr>
<tr>
<td>60.2</td>
<td>69.9</td>
<td>50.9</td>
</tr>
<tr>
<td>60.1</td>
<td>69.8</td>
<td>50.9</td>
</tr>
<tr>
<td>60.2</td>
<td>70.0</td>
<td>50.7</td>
</tr>
<tr>
<td>60.1</td>
<td>69.9</td>
<td>51.0</td>
</tr>
<tr>
<td>60.4</td>
<td>50.9</td>
<td></td>
</tr>
<tr>
<td>60.2</td>
<td>50.9</td>
<td></td>
</tr>
<tr>
<td>60.2</td>
<td>50.8</td>
<td></td>
</tr>
</tbody>
</table>
Reading and Exercises

• Chapter 7
  – Reading pp. 316-48 (2nd ed.)
  – Exercises 2, 5, 6, 7, 14, 15 (2nd ed.)
  – Reading pp. 322-55 (3rd ed.)
  – Exercises 2, 7, 8, 9, 17, 18 (3rd ed.)

The Control Chart for Attributes

Topic
• The Control charts for attributes
• The $p$ and $np$ charts
• Variable sample size
• Sensitivity of the $p$ chart
Types of Data

• Variable data
  • Product characteristic that can be measured
    • Length, size, weight, height, time, velocity

• Attribute data
  • Product characteristic evaluated with a discrete choice
    • Good/bad, yes/no

The Control Chart for Attributes

• In a control chart for variables, quality characteristic is expressed in numbers. Many quality characteristics (e.g., clarity of glass) can be observed only as attributes, i.e., by classifying into defectives and non-defectives.

• If many quality characteristics are measured, a separate control chart for variable will be needed for each quality characteristic. A control chart for attribute is a cheaper alternative. It records an item defective if any specification is not met and non-defective if all the specifications are met.
The Control Chart for Attributes

- The cost of collecting data for attributes is less than that for the variables
- There are various types of control charts for attributes:
  - The $p$ chart for the fraction rejected
  - The $np$ chart for the total number rejected
  - The $c$ chart for the number of defects
  - The $u$ chart for the number of defects per unit

The Control Chart for Attributes

- Poisson Approximation:
  - Occurrence of defectives may be approximated by Poisson distribution
  - Let $n = \text{number of items and } p = \text{proportion of defectives. Then, the expected number of defectives, } \mu_{np} = np$
  - Once the expected number of defectives is known, the probability of $c$ defectives as well as the probability of $c$ or fewer defectives can be obtained from Appendix 4
The Control Chart for Fraction Rejected
The \( p \) Chart: Constant Sample Size

Steps
1. Gather data
2. Calculate \( p \), the proportion of defectives
3. Plot the proportion of defectives on the control chart
4. Calculate the centerline and the control limits (trial)
5. Draw the centerline and control limits on the chart
6. Interpret the chart
7. Revise the chart

The Control Chart for Fraction Rejected
The \( p \) Chart: Constant Sample Size

- Step 4: Calculating trial centerline and control limits for the \( p \) chart

\[
\bar{p} = \frac{\sum np}{\sum n}
\]

\[
UCL_p = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}
\]

\[
LCL_p = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}
\]
The Control Chart for Fraction Rejected
The $p$ Chart: Constant Sample Size

• Step 6: Interpretation of the $p$ chart
  – The interpretation is similar to that of a variable control chart. There should be no patterns in the data such as trends, runs, cycles, or sudden shifts in level. All of the points should fall between the upper and lower control limits.
  – One difference is that for the $p$ chart it is desirable that the points lie near the lower control limit.
  – The process capability is $\bar{p}$, the centerline of the $p$ chart.

The Control Chart for Fraction Rejected
The $p$ Chart: Constant Sample Size

• Step 7: Revised centerline and control limits for the $p$ chart.

\[
\bar{p}_{\text{new}} = \frac{\sum np - np_d}{\sum n - n_d}
\]

\[
UCL_p = \bar{p}_{\text{new}} + 3\sqrt{\frac{p_{\text{new}}(1 - p_{\text{new}})}{\sqrt{n}}}
\]

\[
LCL_p = \bar{p}_{\text{new}} - 3\sqrt{\frac{p_{\text{new}}(1 - p_{\text{new}})}{\sqrt{n}}}
\]
The Control Chart for Fraction Rejected
The \( np \) Chart

- The \( np \) chart construction steps are similar to those of the \( p \) chart. The trial centerline and control limits are as follows:

Centerline \( np = \frac{\sum np}{m} \)

where, \( m \) = number of subgroups

\[ UCL_{np} = np + 3\sqrt{np(1 - p)} \]

\[ LCL_{np} = np - 3\sqrt{np(1 - p)} \]

Variable Sample Size
Choice Between the \( p \) and \( np \) Charts

- If the sample size varies, \( p \) chart is more appropriate
- If the sample size is constant, \( np \) chart may be used
Sensitivity of the $p$ Chart

- Smaller samples are
  - less sensitive to the changes in the quality levels and
  - less satisfactory as an indicator of the assignable causes of variation
- Smaller samples may not be useful at all e.g., if only 0.1% of the product is rejected
- If a control chart is required for a single measurable characteristic, $X$ chart will give useful results with a much smaller sample.

Example 1: A manufacturer purchases small bolts in cartons that usually contain several thousand bolts. Each shipment consists of a number of cartons. As part of the acceptance procedure for these bolts, 400 bolts are selected at random from each carton and are subjected to visual inspection for certain non-conformities. In a shipment of 10 cartons, the respective percentages of rejected bolts in the samples from each carton are 0, 0, 0.5, 0.75, 0.2, 0, 0.25, 0, 0.25, and 1.25. Does this shipment of bolts appear to exhibit statistical control with respect to the quality characteristics examined in this inspection?
Example 2: An item is made in lots of 200 each. The lots are given 100% inspection. The record sheet for the first 25 lots inspected showed that a total of 75 items did not conform to specifications.

a. Determine the trial limits for an np chart.

b. Assume that all points fall within the control limits. What is your estimate of the process average fraction nonconforming $\mu_p$?

c. If this $\mu_p$ remains unchanged, what is the probability that the 26th lot will contain exactly 7 nonconforming units? That it will contain 7 or more nonconforming units? (Hint: use Poisson approximation and Appendix 4)

Example 3: A manufacturer wishes to maintain a process average of 0.5% nonconforming product or less. 1,500 units are produced per day, and 2 days’ runs are combined to form a shipping lot. It is decided to sample 250 units each day and use an np chart to control production.

(a) Find the 3-sigma control limits for this process.

(b) Assume that the process shifts from 0.5 to 4% nonconforming product. Appendix 4 to find the probability that the shift will be detected as the result of the first day’s sampling after the shift occurs.

(c) What is the probability that the shift described in (b) will be caught within the first 3 days after it occurs?
The Control Chart for Attributes

Topic
• The \( p \) chart for the variable sample size
• Calculating \( p \) chart limits using \( n_{ave} \)
• Percent nonconforming chart
• The \( c \) chart
• The \( u \) chart with constant sample size
• The \( u \) chart with variable sample size

The Control Chart for Fraction Rejected
The \( p \) Chart: Variable Sample Size

• When the number of items sampled varies, the \( p \) chart can be easily adapted to varying sample sizes
• If the sample size varies
  – the control limits must be calculated for each different sample size, changing the \( n \) in the control-limit formulas each time a different sample size is taken.
  – calculating the centerline and interpreting the chart will be the same
The Control Chart for Fraction Rejected
The \( p \) Chart: Variable Sample Size

Steps
1. Gather data
2. Calculate \( p \), the proportion of defectives
3. Plot the proportion of defectives on the control chart
4. Calculate the centerline. For each sample calculate a separate pair of control limits.
5. Draw the centerline and control limits on the chart
6. Interpret the chart

\[ p = \frac{\sum_{i=1}^{m} n_i p_i}{\sum_{i=1}^{m} n_i} \] (one centerline for all samples)

\[ UCL_p = \bar{p} + 3 \sqrt{\frac{p(1-p)}{n_i}} \] for the \( i \)-th sample

\[ LCL_p = \bar{p} - 3 \sqrt{\frac{p(1-p)}{n_i}} \] for the \( i \)-th sample
• Step 6: Interpretation of the $p$ chart
  – The interpretation is similar to that of a variable control chart. There should be no patterns in the data such as trends, runs, cycles, or sudden shifts in level. All of the points should fall between the upper and lower control limits.
  – One difference is that for the $p$ chart it is desirable that the points lie near the lower control limit.
  – The process capability is $\bar{p}$, the centerline of the $p$ chart.

The Control Chart for Fraction Rejected
The $p$ Chart: Variable Sample Size

• The calculation of control limits for the $p$ chart with variable sample size can be simplified with the use of $n_{ave}$.

Calculation of Control Limits Using $n_{ave}$

• The value $n_{ave}$ can be found by summing up the individual sample sizes and dividing by the total number of times samples were taken:

$$n_{ave} = \frac{\sum_{i=1}^{m} n_i}{m}$$

where, $m = \text{number of samples}$.
The value \( n_{\text{ave}} \) can be used whenever the individual sample sizes vary no more than 25% from the calculated \( n_{\text{ave}} \).

The advantage of using \( n_{\text{ave}} \) is that there will be a single pair of upper and lower control limits.

\[
UCL_p = \overline{p} + 3 \sqrt{\frac{p(1-p)}{n_{\text{ave}}}}
\]

\[
LCL_p = \overline{p} - 3 \sqrt{\frac{p(1-p)}{n_{\text{ave}}}}
\]

However, if the control limits are computed using the \( n_{\text{ave}} \), the points inside and outside the control limits must be interpreted with caution:

- See the control limit formula - for a larger sample, the control limits are narrower and for a smaller sample, the control limits are wider.
- So, if a larger sample produces a point inside the upper control limit computed using \( n_{\text{ave}} \), the point may actually be outside the upper control limit when the upper control limit is computed using the individual sample size.
The Control Chart for Fraction Rejected
The $p$ Chart: Variable Sample Size
Calculation of Control Limits Using $n_{ave}$

– Similarly, if a smaller sample produces a point outside the upper control limit computed using $n_{ave}$, the point may actually be inside the upper control limit when the upper control limit is computed using the individual sample size

• If a larger sample produces a point inside the upper control limit, the individual control limit should be calculated to see if the process is out-of-control

• If a smaller sample produces a point outside the upper control limit, the individual control limit should be calculated to see if the process is in control

The Control Chart for Fraction Rejected
The $p$ Chart: Variable Sample Size
Calculation of Control Limits Using $n_{ave}$

• The previous discussion leads to the following four cases:

• Case I: The point falls inside the UCL$_p$ and $n_{ind} < n_{ave}$
  – No need to check the individual limit

• Case II: The point falls inside the UCL$_p$ and $n_{ind} > n_{ave}$
  – The individual limits should be calculated

• Case III: The point falls outside the UCL$_p$ and $n_{ind} > n_{ave}$
  – No need to check the individual limit

• Case IV: The point falls outside the UCL$_p$ and $n_{ind} < n_{ave}$
  – The individual limits should be calculated

• **Check points: All the points near UCL. Check only the points which are near UCL.**
The Control Chart for Fraction Rejected
The Percent Nonconforming Chart
Constant Sample Size

- The centerline and control limits for the percent nonconforming chart

\[
\begin{align*}
\text{Centerline } & \quad 100\bar{p} = 100 \sum \frac{np}{n} \\
UCL_{100p} & = 100 \left[ \bar{p} + 3 \sqrt{\frac{p(1-p)}{n}} \right] \\
LCL_{100p} & = 100 \left[ \bar{p} - 3 \sqrt{\frac{p(1-p)}{n}} \right]
\end{align*}
\]

The Control Chart for Nonconformities
The \(c\) and \(u\) charts

- Defective and defect
  - A defective article is the one that fails to conform to some specification.
  - Each instance of the article’s lack of conformity to specifications is a defect
  - A defective article may have one or more defects
The Control Chart for Nonconformities
The $c$ and $u$ charts

- The $np$ and $c$ charts
  - Both the charts apply to total counts
  - The $np$ chart applies to the total number of defectives in samples of constant size
  - The $c$ chart applies to the total number of defects in samples of constant size

- The $p$ and $u$ charts
  - The $p$ chart applies to the proportion of defectives
  - The $u$ chart applies to the number of defects per unit
  - If the sample size varies, the $p$ and $u$ charts may be used

The Control Chart for Counts of Nonconformities
The $c$ Chart: Constant Sample Size

- The number of nonconformities, or $c$, chart is used to track the count of nonconformities observed in a single unit of product of constant size.
- Steps
  1. Gather the data
  2. Count and plot $c$, the count of nonconformities, on the control chart.
  3. Calculate the centerline and the control limits (trial)
  4. Draw the centerline and control limits on the chart
  5. Interpret the chart
  6. Revise the chart
The Control Chart for Counts of Nonconformities
The \( c \) Chart: Constant Sample Size

- Step 3: Calculate the centerline and the control limits (trial)

\[
\text{Centerline } \overline{c} = \frac{\sum c}{m} \\
UCL_c = \overline{c} + 3\sqrt{c} \\
LCL_c = \overline{c} - 3\sqrt{c}
\]

Where, \( m = \) number of samples

The Control Chart for Counts of Nonconformities
The \( c \) Chart: Constant Sample Size

- Step 5: Interpretation of the \( c \) chart
  - The interpretation is similar to that of a variable control chart. There should be no patterns in the data such as trends, runs, cycles, or sudden shifts in level. All of the points should fall between the upper and lower control limits.
  - One difference is that for the \( c \) chart it is desirable that the points lie near the lower control limit.
  - The process capability is \( \overline{c} \), the centerline of the \( c \) chart
The Control Chart for Counts of Nonconformities
The c Chart: Constant Sample Size

• Step 6: Revised centerline and control limits for the c chart

\[
\text{Centerline } c_{\text{new}} = \frac{\sum c - c_d}{m - m_d}
\]

\[
UCL_{c_{\text{new}}} = c_{\text{new}} + 3\sqrt{c_{\text{new}}}
\]

\[
LCL_{c_{\text{new}}} = c_{\text{new}} - 3\sqrt{c_{\text{new}}}
\]

Number of Nonconformities Per Unit
The u Chart: Constant Sample Size

• The number of nonconformities per unit, or u chart, is used to track the number of nonconformities in a unit.

• Steps
  1. Gather the data
  2. Count and plot u, the number of nonconformities per unit, on the control chart.
  3. Calculate the centerline and the control limits (trial)
  4. Draw the centerline and control limits on the chart
  5. Interpret the chart
  6. Revise the chart
• Step 2: Count and plot $u$, the number of nonconformities per unit, on the control chart.

Let

$n = \text{number of inspected items in a sample}$
$c = \text{number of nonconformities in a sample}$

$$u = \frac{c}{n}$$

---

• Step 3: Calculate the centerline and the control limits (trial)

Centerline

$$\bar{u} = \frac{\sum c}{\sum n}$$

$$UCL_u = \bar{u} + 3\frac{\sqrt{\bar{u}}}{\sqrt{n}}$$

$$LCL_u = \bar{u} - 3\frac{\sqrt{\bar{u}}}{\sqrt{n}}$$
Number of Nonconformities Per Unit  
The \( u \) Chart: Constant Sample Size

• Step 6: Revise the chart

\[
\begin{align*}
\text{Centerline } \bar{u} &= \frac{\sum c - c_d}{\sum n - n_d} \\
UCL_{u_{\text{ave}}} &= \bar{u}_{\text{new}} + 3\sqrt{\frac{\bar{u}_{\text{new}}}{\sqrt{n}}} \\
LCL_{u_{\text{ave}}} &= \bar{u}_{\text{new}} - 3\sqrt{\frac{\bar{u}_{\text{new}}}{\sqrt{n}}}
\end{align*}
\]

Number of Nonconformities Per Unit  
The \( u \) Chart: Variable Sample Size

• When the sample size varies, compute
  – either the individual control limits
  – or a control limit using \( n_{\text{ave}} \)

\[
\begin{align*}
n_{\text{ave}} &= \frac{\sum_{i=1}^{m} n_i}{m} \quad \text{(trial)} \quad \text{or} \quad n_{\text{ave}} &= \frac{\sum_{i=1}^{m} n_i - n_d}{m - m_d} \quad \text{(revised)}
\end{align*}
\]

where, \( m = \text{number of samples} \)

• When using \( n_{\text{ave}} \) no individual sample size may vary more than 25% from \( n_{\text{ave}} \)
The Control Chart for Nonconformities
The c and u charts

• As the Poisson distribution is not symmetrical, the upper and lower 3-sigma limits do not correspond to equal probabilities of a point on the control chart falling outside limits. To avoid the problem with asymmetry, the use of 0.995 and 0.005 limits has been favored

• If the distribution does not follow Poisson law, actual standard deviation may be greater than $\sqrt{c}$ and, therefore, 3-sigma limit may actually be greater than $3\sqrt{c}$ limit obtained from the formula

---

Example 4: The following are data on 5-gal containers of paint. If the color mixture of the paint does not match the control color, then the entire container is considered nonconforming and is disposed of. Since the amount produced during each production run varies, use $n_{ave}$ to calculate the centerline and control limits for this set of data. Carry calculations to four decimal places. Remember to Round $n_{ave}$ to a whole number; you can’t sample part of a 5-gal pail.

<table>
<thead>
<tr>
<th>Production run</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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</thead>
<tbody>
<tr>
<td>Number inspected</td>
<td>2,524</td>
<td>2,056</td>
<td>2,750</td>
<td>3,069</td>
<td>3,365</td>
<td>3,763</td>
</tr>
<tr>
<td>Number defective</td>
<td>30</td>
<td>84</td>
<td>76</td>
<td>108</td>
<td>54</td>
<td>29</td>
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<table>
<thead>
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<th>Production run</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
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<td>2,255</td>
<td>2,060</td>
<td>2,835</td>
<td>2,620</td>
<td>2,250</td>
</tr>
<tr>
<td>Number defective</td>
<td>20</td>
<td>25</td>
<td>48</td>
<td>10</td>
<td>86</td>
<td>25</td>
</tr>
<tr>
<td>Production Run</td>
<td>Number Inspected</td>
<td>Number Defective</td>
<td>Proportion Defective</td>
<td>Check Point?</td>
<td></td>
<td></td>
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<td>------------------</td>
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</tr>
<tr>
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<tr>
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<tr>
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<td>2,750</td>
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<td>6</td>
<td>3,763</td>
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<tr>
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<td>2,675</td>
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<td>2,255</td>
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<td>9</td>
<td>2,060</td>
<td>48</td>
<td>0.0233</td>
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<tr>
<td>10</td>
<td>2,835</td>
<td>10</td>
<td>0.0035</td>
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<tr>
<td>11</td>
<td>2,620</td>
<td>86</td>
<td>0.0328</td>
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<td>12</td>
<td>2,250</td>
<td>25</td>
<td>0.0111</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Example 5:** A $c$ chart is used to monitor the number of surface imperfections on sheets of photographic film. The chart presently is set up based on $c$ of 2.6.

(a) Find the 3-sigma control limits for this process.

(b) Use Appendix 4 to determine the probability that a point will fall outside these control limits while the process is actually operating at a $\mu_c$ of 2.6.

(c) If the process average shifts to 4.8, what is the probability of not detecting the shift on the first sample taken after the shift occurs?
Example 6: A shop uses a control chart on maintenance workers based on maintenance errors per standard worker-hour. For each worker, a random sample of 5 items is taken daily and the statistic \( c/n \) is plotted on the worker’s control chart where \( c \) is the count of errors found in 5 assemblies and \( n \) is the total worker-hours required for the 5 assemblies.

(a) After the first 4 weeks, the record for one worker is \( \sum c = 22 \) and \( \sum n = 54 \). Determine the central line and the 3-sigma control limits.

(b) On a certain day during the 4-week period, the worker makes 2 errors in 4.3 standard worker-hour. Determine if the point for this day falls within control limits.

Reading and Exercises

- Chapter 9:
  - Reading pp. 404-447 (2nd ed.)
  - Problems 5, 10 (solve with and without \( \bar{n} \)), 11, 13, 14, 19, 20, 23, 25 (2nd ed.)
  - Reading pp. 414-453 (3rd ed.)
  - Problems 5, 10 (solve with and without \( \bar{n} \)), 11, 13, 14, 19, 20, 23, 25 (2nd ed.)
Acceptance Sampling

Outline

• Sampling
• Some sampling plans
• A single sampling plan
• Some definitions
• Operating characteristic curve

Necessity of Sampling

• In most cases 100% inspection is too costly.
• In some cases 100% inspection may be impossible.
• If only the defective items are returned, repair or replacement may be cheaper than improving quality. But, if the entire lot is returned on the basis of sample quality, then the producer has a much greater motivation to improve quality.
Some Sampling Plans

- Single sampling plans:
  - Most popular and easiest to use
  - Two numbers $n$ and $c$
  - If there are more than $c$ defectives in a sample of size $n$ the lot is rejected; otherwise it is accepted

- Double sampling plans:
  - A sample of size $n_1$ is selected.
  - If the number of defectives is less than or equal to $c_1$ than the lot is accepted.
  - Else, another sample of size $n_2$ is drawn.
  - If the cumulative number of defectives in both samples is more than $c_2$ the lot is rejected; otherwise it is accepted.

Some Sampling Plans

- A double sampling plan is associated with four numbers:
  $n_1, n_2, c_1$ and $c_2$

- The interpretation of the numbers is shown by an example:
  Let $n_1 = 20, n_2 = 10, c_1 = 3, c_2 = 5$
  1. Inspect a sample of size 20
  2. If the sample contains 3 or less defectives, accept the lot
  3. If the sample contains more than 5 defectives, reject the lot.
Some Sampling Plans

4. If the sample contains more than 3 and less than or equal to 5 defectives (i.e., 4 or 5 defectives), then inspect a second sample of size 10.

5. If the cumulative number of defectives in the combined sample of 30 is not more than 5, then accept the lot.

6. Reject the lot if there are more than 5 defectives in the combined lot of 30.

• Double sampling plans may be extended to triple sampling plans, which may also be extended to higher order plans. The logical conclusion of this process is the multiple or sequential sampling plan.

Some Sampling Plans

• Multiple sampling plans
  – The decisions (regarding accept/reject/continue) are made after each lot is sampled.
  – A finite number of samples (at least 3) are taken.

• Sequential sampling plans
  – Items are sampled one at a time and the cumulative number of defectives is recorded at each stage of the process.
  – Based on the value of the cumulative number of defectives there are three possible decisions at each stage:
    • Reject the lot
    • Accept the lot
    • Continue sampling
Some Sampling Plans

• Multiple sampling and sequential sampling are very similar. Usually, in a multiple sampling plan the decisions (regarding accept/reject/continue) are made after each lot is sampled. On the other hand, in a sequential sampling plan, the decisions are made after each item is sampled. In a multiple sampling, a finite number of samples (at least 3) are taken. A sequential sampling may not have any limit on the number of items inspected.

Some Definitions

• Acceptable quality level (AQL)
  Acceptable fraction defective in a lot
• Lot tolerance percent defective (LTPD)
  Maximum fraction defective accepted in a lot
• Producer’s risk, $\alpha$
  Type I error = $P(\text{reject a lot}|\text{probability(defective)=AQL})$
• Consumer’s risk, $\beta$
  Type II error = $P(\text{accept a lot}|\text{probability(defective)=LTPD})$
A Single Sampling Plan

Consider a single sampling plan with \( n = 10 \) and \( c = 2 \)

- Compute the probability that a lot will be accepted with a proportion of defectives, \( p = 0.10 \)

- If a producer wants a lot with \( p = 0.10 \) to be accepted, the sampling plan has a risk of _______________
- This is producer’s risk, \( \alpha \) and AQL = 0.10

A Single Sampling Plan

- Compute the probability that a lot will be accepted with a proportion of defectives, \( p = 0.30 \)

- If a consumer wants to reject a lot with \( p = 0.30 \), the sampling plan has a risk of _______________
- This is consumer’s risk, \( \beta \) and LTPD = 0.30
Approximation to Binomial Distribution

Under some circumstances, it may be desirable to obtain $\alpha$ and $\beta$ by an approximation of binomial distribution

- Poisson distribution: When $p$ is small and $n$ is moderately large ($n>25$ and $np<5$)

- Normal distribution: When $n$ is very large, $np(1-p)>5$

Example: Samples of size 50 are drawn from lots 200 items and the lots are rejected if the number of defectives in the sample exceeds 4. If the true proportion of defectives in the lot is 10 percent, determine the probability that a lot is accepted using

a. The Poisson approximation to the binomial

b. The normal approximation to the binomial
**Example:** Samples of size 50 are drawn from lots 200 items and the lots are rejected if the number of defectives in the sample exceeds 4. If the true proportion of defectives in the lot is 10 percent, determine the probability that a lot is accepted using

a. The Poisson approximation to the binomial

\[ \lambda = np = 50(0.10) = 5 \]

\[ P\{X \leq 4 \mid \lambda = 5\} = 1 - 0.5595 \text{ (Table A - 3)} = 0.4405 \]

b. The normal approximation to the binomial

\[ \mu = np = 50(0.10) = 5 \]

\[ \sigma = \sqrt{np(1 - p)} = \sqrt{50(0.1)(0.90)} = 2.12 \]

\[ P\{X \leq 4\} = P\left\{ z \leq \frac{4.5 - 5}{2.12} \right\} = P\{z \leq -0.2357\} \]

\[ = 0.5 - 0.0948 \text{ (Table A - 1)} = 0.4052 \]

---

**Operating Characteristic Curve**

![OC curve for n and c](image)

- **AQL**
- **LTPD**
- **Percent defective**
- **Probability of acceptance, P_a**
- **Probability of non-acceptance, P_b**
- **α = 0.05**
- **β = 0.10**
Operating Characteristic Curve

OC Curve by Poisson Approximation

<table>
<thead>
<tr>
<th>n</th>
<th>100</th>
<th>( \alpha )</th>
</tr>
</thead>
<tbody>
<tr>
<td>c</td>
<td>4</td>
<td>0.052653017</td>
</tr>
<tr>
<td>AQL</td>
<td>0.02</td>
<td>( \beta )</td>
</tr>
<tr>
<td>LTPD</td>
<td>0.08</td>
<td>0.0996324</td>
</tr>
</tbody>
</table>

| Defective | Probability of c or less | Defective | Probability of defects
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( p )</td>
<td>np</td>
<td>( Pa )</td>
<td></td>
</tr>
<tr>
<td>0.005</td>
<td>0.5</td>
<td>0.999827884</td>
<td></td>
</tr>
<tr>
<td>0.01</td>
<td>1</td>
<td>0.996340153</td>
<td></td>
</tr>
<tr>
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</tr>
<tr>
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</tr>
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</tr>
<tr>
<td>0.03</td>
<td>3</td>
<td>0.815263245</td>
<td></td>
</tr>
</tbody>
</table>

Operating Characteristics Curve
OC Curve of an Ideal Sampling Plan

• Suppose that 2% is the maximum tolerable proportion defective in a lot
• So, an ideal sampling scheme would reject all lots that were worse than 2% defective and accepted all lots 2% defective better
• The OC Curve of such an ideal scheme would be vertical at \( p=0.02 \)
• However, no sampling plan can give such an ideal OC curve

Effect of Changing the Sampling Plan

• The larger the sample size, the steeper the slope of the OC Curve
  – Note that this statement is true if both \( n \) and \( c \) are increased proportionately.
• If only \( n \) increases, every \( P_a \) decreases and the curve shifts downward - so, producer’s risk increases and consumer’s risk decreases
• If only \( c \) increases, every \( P_a \) increases and the curve shifts upward - so, producer’s risk decreases and consumer’s risk increases
Reading

• Acceptance Sampling

Average Outgoing Quality

Outline

• Average Outgoing Quality (AOQ)
• Average Outgoing Quality Limit (AOQL)
Average Outgoing Quality

• After a sample is inspected, the items which are found defective, may be
  – Case 1: returned to the producer or
  – Case 2: repaired or replaced by the producer.
• We assume Case (2).
• If a lot is rejected, it may be subjected to a 100% inspection. Such action is referred to as screening inspection, or detailing. This is sometimes described as an acceptance/rectification scheme.

Average Outgoing Quality

• If a lot is rejected, there may again be two assumptions regarding the defective items. The defective items may be
  – Case 1: returned to the producer or
  – Case 2: repaired or replaced by the producer.
• We assume Case 2.
• So, if a lot is rejected, it will contain no defective item at all. The consumer will get $N$ good items. However, if a lot is accepted, it may contain some defective items because many of the $(N-n)$ items in a single sampling plan) items not inspected may be defective.
Average Outgoing Quality

- Thus, if there is an average of 2% defective items, the accepted lots will contain little less than 2% defective items and rejected lots will contain no defective item at all. On average, the consumer will receive less than 2% defective items.
- Given a proportion of defective, $p$ the Average Outgoing Quality (AOQ) is the proportion of defectives items in the outgoing lots. More precise definition is given in the next slide.

Average Outgoing Quality

$$\text{AOQ} = \frac{E\{\text{Outgoing number of defectives}\}}{E\{\text{Outgoing number of items}\}}$$

Let

$P_a = P\{\text{lot is accepted} | \text{proportion of defectives} = p\}$

$N = \text{Number of items in the lot}$

$n = \text{Number of items in the sample}$
Average Outgoing Quality

Case 1 is not discussed in class

Case 1: Defective items are not replaced

\[
AOQ = \frac{P_a(N-n)p}{N-np - p(1-P_a)(N-n)}
\]

If \(N\) is much larger than \(n\),

\[
AOQ = \frac{P_a p}{1 - p(1-P_a)}
\]

Average Outgoing Quality

Case 2: Defective items are replaced

\[
AOQ = \frac{P_a(N-n)p}{N}
\]

If \(N\) is much larger than \(n\),

\[
AOQ = P_a p
\]
Average Outgoing Quality

- Given a proportion of defective, we can compute the Average Outgoing Quality (AOQ)
- As \( p \) increases from 0.0, the AOQ values increases up to a limit called Average Outgoing Quality Limit (AOQL), after which the AOQ values descend continuously to 0.0. This is shown in the next slide.
Example: Suppose that Noise King is using rectified inspection for its single sampling plan. Calculate the average outgoing quality limit for a plan with \( n = 110 \), \( c = 3 \), and \( N = 1000 \). (Assume that the defective items are replaced)

<table>
<thead>
<tr>
<th>( n )</th>
<th>110</th>
<th>( c )</th>
<th>3</th>
<th>( N )</th>
<th>1000</th>
<th>AOQL</th>
<th>0.01564264</th>
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</thead>
</table>

<table>
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<tr>
<th>Probability</th>
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<th>Defective</th>
<th>Defects</th>
<th>( p )</th>
<th>np</th>
<th>(Pa)</th>
<th>AOQ</th>
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<tbody>
<tr>
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</table>
Reading

- Average Outgoing Quality
CHAPTER 10
RELIABILITY

- Failure rates
- Reliability
- Constant failure rate and exponential distribution
- System Reliability
  - Components in series
  - Components in parallel
  - Combination system

Failure Rate Curve

- Early failure
  a.k.a.
  Infant
  mortality
  period

- Normal
  operating
  period

- Wearout
  period

Time
Reliability

- Reliability provides a numerical measure of “degree of excellence” through time.
  - Failure: the inability of an equipment to perform its required function
  - Reliability: the probability of no failure throughout a prescribed operating period.

Constant Failure Rate Assumption and the Exponential Distribution

Example 1: Suppose that there is a 0.001 probability that a light bulb will fail in one hour. What is the probability that the light bulb will survive

a. 2 hours

b. 3 hours

c. 1000 hours
Constant Failure Rate Assumption and the Exponential Distribution

Example 2: Suppose that the probability that a light bulb will fail in one hour is $\lambda$. What is the probability that the light bulb will survive at least $t$ hours?

a. Use conditional probabilities (as in Example 1)

b. Use Exponential distribution

<table>
<thead>
<tr>
<th>Age, $t$</th>
<th>$(1-\lambda)^t$</th>
<th>$\exp(-\lambda t)$</th>
<th>difference</th>
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<td>1</td>
<td>0</td>
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<tr>
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<td>0.001</td>
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</table>

Justification of the use of Exponential Distribution to estimate reliability

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<th>Failure rate, $\lambda$</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Maximum difference</td>
<td>0.004</td>
</tr>
</tbody>
</table>

Probability of survival at least $t$ periods
Constant Failure Rate Assumption and the Exponential Distribution

- The difference of probabilities computed in two methods (conditional probabilities and exponential distribution) is small for
  - small values of $\lambda$ and
  - large values of $t$
- The use of conditional probabilities is correct for an artificial experiment in which $t$ can be only integers
- Exponential distribution is widely used to represent the constant failure rate
- Weibull distribution is used to represent increasing/constant/decreasing failure rates

Failure Rate and Average Life

- Failure rate, $\lambda$, the probability of a failure during a stated period is calculated as follows:
  $$\lambda = \frac{\text{number of failures observed}}{\text{sum of test times or cycles}}$$
- The average life, $\theta$ is calculated as follows:
  $$\theta = \frac{\text{sum of test times or cycles}}{\text{number of failures observed}} = \frac{1}{\lambda}$$
Failure Rate and Average Life

Example 3: Determine the failure rate for a 80-hour test of 10 items where 2 items fail at 40 and 70 hours, respectively. What is the mean life of the product?

Availability

- The average life, $\theta$ is also called as
  - Mean Time Between Failure (MTBF) if the system is repairable
  - Mean Time To Failure (MTTF) if the system is non-repairable

$$\text{Availability} = \frac{\text{MTBF}}{\text{MTBF} + \text{Mean Time To Repair (MTTR)}}$$
Availability

Example 4: A copier machine has a mean time between failures of 475 operating hours. Repairs typically require an average of 25 hours from the time that the repair call is received until service is completed. Determine the availability of the copier.

An OC Curve For An Acceptance Sampling Plan Based on Mean Life

• Previously, we discussed an OC curve that
  – corresponds to a given sampling plan, say $n$ and $c$ and
  – shows probability of acceptance as a function of proportion of defective items
An OC Curve For An Acceptance Sampling Plan Based on Mean Life

• Similarly,
  – An OC curve may be constructed showing the probability of acceptance as a function of average life, \( \theta \)
  – In this case, the sampling plan may be defined with
    • Number of hours of test and
    • an acceptance number
  – A major assumption
    • The failed item will be replaced by a good item.

1. Consider a sampling plan with
   – Number of hours of test, \( T \) and
   – an acceptance number, \( c \)

2. For each average life, \( \theta \),
   – Compute the failure rate per hour \( \lambda = \frac{1}{\theta} \)
   – Compute the expected number of failures during the test \( \mu_c = \lambda T \)
   – Compute \( P(\text{acceptance}) = P(\text{c or fewer failure}) = 1 - P(\text{c+1 or more failure when the mean number of failures is } \mu_c) \) this can be obtained from using Poisson probabilities given in Table A-3
Example 5: In one of the plans, 10 items were to be tested for 500 h with replacement and with an acceptance number of 1. Plot an OC curve showing probability of acceptance as a function of average life.

An OC Curve For An Acceptance Sampling Plan Based on Mean Life

<table>
<thead>
<tr>
<th>Mean life (θ)</th>
<th>Failure Rate</th>
<th>Expected Average Number of Failure</th>
<th>Probability of Acceptance (Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0.01</td>
<td>50</td>
<td>9.8366E-21</td>
</tr>
<tr>
<td>200</td>
<td>0.005</td>
<td>25</td>
<td>3.6109E-10</td>
</tr>
<tr>
<td>300</td>
<td>0.003333333</td>
<td>16.66667</td>
<td>1.0207E-06</td>
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<tr>
<td>400</td>
<td>0.0025</td>
<td>12.5</td>
<td>5.031E-05</td>
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<tr>
<td>500</td>
<td>0.002</td>
<td>10</td>
<td>0.0004994</td>
</tr>
<tr>
<td>600</td>
<td>0.001666667</td>
<td>8.333333</td>
<td>0.00224345</td>
</tr>
</tbody>
</table>
**System Reliability**

- Most products are made up of a number of components
- The reliability of each component and the configuration of the system consisting of these components determines the system reliability (i.e., the reliability of the product).
- The components may be in
  - series: system operates if all components operate
  - parallel: system operates is any component operates
  - combination of series and parallel
Components in Series

- If the components are in series, the system operates if all the components operate.
- If there are $n$ components in series, where the reliability of the $i$-th component is denoted by $r_i$, the system reliability is

$$R_s = r_1 \cdot r_2 \cdots r_n$$

**Example 6:** A module of a satellite monitoring system has 500 components in series. The reliability of each component is 0.999. Find the reliability of the module. If the number of components is reduced to 200, what is the reliability?

With 500 components, $R_s = r_1 \cdot r_2 \cdots r_{500} = (0.999)^{500} = 0.606$

With 200 components, $R_s = r_1 \cdot r_2 \cdots r_{200} = (0.999)^{200} = 0.819$
Components in Parallel

- If the components are in parallel, the system operates if any component operates.
- If there are $n$ components in parallel, where the reliability of the $i$-th component is denoted by $r_i$, the system reliability is

$$R_p = 1 - (1 - r_1)(1 - r_2) \cdots (1 - r_n)$$

Example 7: Find the reliability of a system with three components, A, B, and C in parallel. The reliabilities of A, B, and C are 0.95, 0.92, and 0.90, respectively.

$$R_p = 1 - (1 - r_1)(1 - r_2)(1 - r_3)$$

$$= 1 - (1 - 0.95)(1 - 0.92)(1 - 0.90)$$

$$= 1 - 0.0004 = 0.9996$$
Redundant Systems and Backup Components

- If a system contains a backup or spare components, it can be treated as the one with components in parallel. The following formula

\[ R_b = r_i + r_b(1 - r_i) \]

is equivalent to

\[ R_p = 1 - (1 - r_i)(1 - r_b) \]

Combination System

**Example 8:** Find the reliability of the following system
Reading and Exercises

• Chapter 10:
  – Reading pp. 478-495, Problems 4,5,8 (2nd ed.)
  – Reading pp. 484-501, Problems 4,5,8 (3rd ed.)