CHAPTER 10
RELIABILITY

- Failure rates
- Reliability
- Constant failure rate and exponential distribution
- System Reliability
  - Components in series
  - Components in parallel
  - Combination system

Failure Rate Curve

<table>
<thead>
<tr>
<th>Failure rate</th>
<th>Early failure a.k.a. Infant mortality period</th>
<th>Normal operating period</th>
<th>Wearout period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Reliability

- Reliability provides a numerical measure of “degree of excellence” through time.
  - Failure: the inability of an equipment to perform its required function
  - Reliability: the probability of no failure throughout a prescribed operating period.

Constant Failure Rate Assumption and the Exponential Distribution

Example 1: Suppose that there is a 0.001 probability that a light bulb will fail in one hour. What is the probability that the light bulb will survive

a. 2 hours
b. 3 hours
c. 1000 hours
Constant Failure Rate Assumption and the Exponential Distribution

Example 2: Suppose that the probability that a light bulb will fail in one hour is $\lambda$. What is the probability that the light bulb will survive at least $t$ hours?

a. Use conditional probabilities (as in Example 1)

b. Use Exponential distribution

<table>
<thead>
<tr>
<th>Age, $t$</th>
<th>$(1-\lambda)^t$</th>
<th>$\exp(-\lambda t)$</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0.980</td>
<td>0.980</td>
<td>0.000</td>
</tr>
<tr>
<td>2</td>
<td>0.960</td>
<td>0.961</td>
<td>0.000</td>
</tr>
<tr>
<td>3</td>
<td>0.941</td>
<td>0.942</td>
<td>0.001</td>
</tr>
<tr>
<td>4</td>
<td>0.922</td>
<td>0.923</td>
<td>0.001</td>
</tr>
<tr>
<td>5</td>
<td>0.904</td>
<td>0.905</td>
<td>0.001</td>
</tr>
</tbody>
</table>

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Constant Failure Rate Assumption and the Exponential Distribution

• The difference of probabilities computed in two methods (conditional probabilities and exponential distribution) is small for
  – small values of $\lambda$ and
  – large values of $t$
• The use of conditional probabilities is correct for an artificial experiment in which $t$ can be only integers
• Exponential distribution is widely used to represent the constant failure rate
• Weibull distribution is used to represent increasing/constant/decreasing failure rates

Failure Rate and Average Life

• Failure rate, $\lambda$, the probability of a failure during a stated period is calculated as follows:
  \[ \lambda = \frac{\text{number of failures observed}}{\text{sum of test times or cycles}} \]
• The average life, $\theta$ is calculated as follows:
  \[ \theta = \frac{\text{sum of test times or cycles}}{\text{number of failures observed}} = \frac{1}{\lambda} \]
Failure Rate and Average Life

Example 3: Determine the failure rate for a 80-hour test of 10 items where 2 items fail at 40 and 70 hours, respectively. What is the mean life of the product?

Availability

- The average life, \( \theta \) is also called as
  - Mean Time Between Failure (MTBF) if the system is repairable
  - Mean Time To Failure (MTTF) if the system is non-repairable

\[
\text{Availability} = \frac{\text{MTBF}}{\text{MTBF} + \text{Mean Time To Repair (MTTR)}}
\]
Availability

Example 4: A copier machine has a mean time between failures of 475 operating hours. Repairs typically require an average of 25 hours from the time that the repair call is received until service is completed. Determine the availability of the copier.

An OC Curve For An Acceptance Sampling Plan Based on Mean Life

• Previously, we discussed an OC curve that
  – corresponds to a given sampling plan, say $n$ and $c$ and
  – shows probability of acceptance as a function of proportion of defective items
An OC Curve For An Acceptance Sampling Plan Based on Mean Life

• Similarly,
  – An OC curve may be constructed showing the probability of acceptance as a function of average life, $\theta$
  – In this case, the sampling plan may be defined with
    • Number of hours of test and
    • an acceptance number
  – A major assumption
    • The failed item will be replaced by a good item.

An OC Curve For An Acceptance Sampling Plan Based on Mean Life

1. Consider a sampling plan with
   – Number of hours of test, $T$ and
   – an acceptance number, $c$
2. For each average life, $\theta$,
   – Compute the failure rate per hour $\lambda = \frac{1}{\theta}$
   – Compute the expected number of failures during the test $\mu_c = \lambda T$
   – Compute $P(\text{acceptance}) = P(c \text{ or fewer failure}) = 1 - P(c+1 \text{ or more failure when the mean number of failures is } \mu_c)$
     this can be obtained from using Poisson probabilities given in Table A-3
Example 5: In one of the plans, 10 items were to be tested for 500 h with replacement and with an acceptance number of 1. Plot an OC curve showing probability of acceptance as a function of average life.

<table>
<thead>
<tr>
<th>Mean life (θ)</th>
<th>Failure Rate (λ = 1/θ)</th>
<th>Expected Number of Failure (μc = λT)</th>
<th>Probability of Acceptance (Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0.01</td>
<td>50</td>
<td>9.8366E-21</td>
</tr>
<tr>
<td>200</td>
<td>0.005</td>
<td>25</td>
<td>3.6109E-10</td>
</tr>
<tr>
<td>300</td>
<td>0.003333333</td>
<td>16.66667</td>
<td>1.0207E-06</td>
</tr>
<tr>
<td>400</td>
<td>0.0025</td>
<td>12.5</td>
<td>5.031E-05</td>
</tr>
<tr>
<td>500</td>
<td>0.002</td>
<td>10</td>
<td>0.0004994</td>
</tr>
<tr>
<td>600</td>
<td>0.001666667</td>
<td>8.333333</td>
<td>0.00224345</td>
</tr>
</tbody>
</table>
System Reliability

- Most products are made up of a number of components
- The reliability of each component and the configuration of the system consisting of these components determines the system reliability (i.e., the reliability of the product).
- The components may be in
  - series: system operates if all components operate
  - parallel: system operates if any component operates
  - combination of series and parallel
Components in Series

- If the components are in series, the system operates if all the components operate
- If there are \( n \) components in series, where the reliability of the \( i \)-th component is denoted by \( r_i \), the system reliability is

\[
R_s = \left( r_1 \right) \left( r_2 \right) \cdots \left( r_n \right)
\]

Example 6: A module of a satellite monitoring system has 500 components in series. The reliability of each component is 0.999. Find the reliability of the module. If the number of components is reduced to 200, what is the reliability?

With 500 components, \( R_s = \left( r_1 \right) \left( r_2 \right) \cdots \left( r_{500} \right) = (0.999)^{500} = 0.606 \)

With 200 components, \( R_s = \left( r_1 \right) \left( r_2 \right) \cdots \left( r_{200} \right) = (0.999)^{200} = 0.819 \)
Components in Parallel

- If the components are in parallel, the system operates if any component operates.
- If there are $n$ components in parallel, where the reliability of the $i$-th component is denoted by $r_i$, the system reliability is

$$ R_p = 1 - (1 - r_1)(1 - r_2)\cdots(1 - r_n) $$

**Example 7:** Find the reliability of a system with three components, A, B, and C in parallel. The reliabilities of A, B, and C are 0.95, 0.92, and 0.90, respectively.

$$ R_p = 1 - (1 - r_1)(1 - r_2)(1 - r_3) $$

$$ = 1 - (1 - 0.95)(1 - 0.92)(1 - 0.90) $$

$$ = 1 - 0.0004 = 0.9996 $$
Redundant Systems and Backup Components

- If a system contains a backup or spare components, it can be treated as the one with components in parallel. The following formula

\[ R_b = r_i + r_b(1 - r_i) \]

is equivalent to

\[ R_p = 1 - (1 - r_i)(1 - r_b) \]

Combination System

Example 8: Find the reliability of the following system

![Combination System Diagram]

\[ 0.89 \]
\[ 0.99 \]
\[ 0.89 \]
\[ 0.98 \]
\[ 0.95 \]
\[ 0.95 \]
Reading and Exercises

• Chapter 10:
  – Reading pp. 478-495, Problems 4,5,8 (2\textsuperscript{nd} ed.)
  – Reading pp. 484-501, Problems 4,5,8 (3\textsuperscript{rd} ed.)