

De Broglie Waves as an Effect of Clock Desynchronization

William E. Baylis

Department of Physics, University of Windsor
Windsor, ON, Canada N9B 3P4

August 2, 2007

Abstract

De Broglie waves are a simple consequence of special relativity applied to the complex-phase oscillations of stationary states. As de Broglie showed in his doctoral thesis, the synchronized oscillations of an extended system at rest, even a classical one, become de Broglie-like waves when boosted to finite velocity. The waves illustrate the well-known but seldom demonstrated relativistic effect of clock desynchronization (or dephasing) in moving frames. Although common manifestations of stationary-state oscillations in interference experiments are sensitive only to energy differences, de Broglie wavelengths are inversely proportional to rest-frame oscillation frequency, and their observed values require that the oscillation frequencies are proportional to the the total absolute energy, including the rest component mc^2 .

1 Introduction

The wave nature of matter as predicted by Louis de Broglie in his doctoral dissertation of 1924[1, 2] has formed an accepted cornerstone of quantum mechanics ever since electrons[3, 4] and later neutrons[5] were diffracted by crystals and thin films. Diffraction and interferometry has also confirmed the de Broglie wavelength relation for atoms[6, 7, 8, 9, 10, 11] and molecules as large as C_{60} and C_{70} fullerenes[12, 13]. The advent of Bose-Einstein condensates has provided new tools for studying matter waves, such as laser-like sources of atoms[14] to enhance observations of diffraction and interference in atomic systems.

As well documented, de Broglie based his suggestion of matter waves on an analogy with the wave-particle duality in electromagnetic waves/photons. His proposal[15] was that a free particle of mass m is subject to a periodic oscillation of frequency $\nu_0 = mc^2/h$ in its rest frame, where $h = 2\pi\hbar$ is Planck's constant and c is the speed of light. The phase of the oscillation in the rest frame can be written $2\pi\nu_0\tau \equiv \omega_0\tau$, where τ is the time in the rest frame, also called the proper time of the particle. By application of a Lorentz boost (velocity transformation),

de Broglie showed[1, 2] that the phase can be expressed in coordinates (t, \mathbf{x}) of the lab frame, where the particle moves with velocity \mathbf{v} , by

$$\omega_0\tau = \gamma\omega_0 \left(t - \mathbf{v} \cdot \mathbf{x}/c^2 \right),$$

with the Lorentz factor $\gamma = (1 - v^2/c^2)^{-1/2}$. As discussed in more detail below, the result is a wave of phase $\omega t - \mathbf{k} \cdot \mathbf{x}$ whose spacetime wave vector $k = e_0\omega/c + \mathbf{k}$ is proportional to the spacetime momentum $p = Ee_0/c + \mathbf{p}$ of the massive particle:

$$p = \hbar k. \tag{1}$$

Here e_0 is unit displacement on the time axis. Relation (1) is identical to that relating the momentum p of a photon to the wave vector k of its associated electromagnetic wave.

Note that p and k are vectors in a four-dimensional Minkowski spacetime. They are sometimes called “4-vectors”, “world vectors”, or “paravectors”[16] to distinguish them from common spatial vectors in three-dimensional physical space. Relation (1) thus implies an equality both of time components along e_0 , giving $E = \hbar\omega$, and of spatial components along the spatial axes e_1, e_2, e_3 , giving $\mathbf{p} = \hbar\mathbf{k}$. The spatial part gives the de Broglie wavelength $\lambda = 2\pi/|\mathbf{k}| = h/|\mathbf{p}|$, with all its well-verified physical implications.

The phase velocities of waves for massive particles are superluminal: $\omega/|\mathbf{k}| = E/|\mathbf{p}| > c$, and this caused de Broglie initially to regard these waves as “fictitious.”[15] He showed that the actual particle velocity is given by the subluminal group velocity $d\omega/d|\mathbf{k}| = c^2|\mathbf{k}|/\omega$ of the wave, and that if the rest-frame oscillations are synchronized over an extended region in the rest frame, the phase velocity must be $> c$ to keep in step with the moving particle in the lab.

De Broglie’s work spawned a revolution in quantum theory. A linear superposition of de Broglie waves gives the wave function of Schrödinger wave mechanics.[17] De Broglie’s approach[1] relating Hamilton’s principle and the principle of least action for quantum waves the way Fermat’s principle relates wave and geometric optics also presaged the path-integral formulation of quantum theory[18] and Feynman’s popular presentation of it.[19] It may also be of historical interest that de Broglie, searching for a more physical interpretation of his waves, developed his “double solution theory”[20, 21], in which the superluminal waves act to guide the particle in a manner closely related to the causal formulation of quantum mechanics developed by Bohm in his deterministic non-local (“hidden-variable”) theory.[22, 23]

This communication points out that any in-phase oscillations of an extended system in the rest frame imply de Broglie-like waves in other inertial frames. The waves arise from the essential relativistic property that time is not universal. This property has the effect of dephasing or desynchronizing the clocks in a moving frame. The effect is required in order that phenomena such as time dilation and Lorentz contraction depend only on relative velocity. It causes clocks at different positions that all read the same time in the rest frame to read different times when viewed at an instant from a frame in relative motion. While in his Ph.D. thesis[1], de Broglie left unspecified the physical nature of

the underlying oscillations, he drew spacetime diagrams similar to one below to illustrate the phenomena. We can now associate the rest-frame oscillations with the complex phase of any stationary state. De Broglie waves are a necessary consequence of such oscillations and special relativity.

The explanation, although hardly ever mentioned in texts, gives a simple picture of the superluminal phase wave and provides an important example of clock desynchronization and the way that simultaneity depends on the observer frame. It is also noteworthy that the wave has the correct de Broglie wavelength if and only if the oscillation frequency of the stationary state times the Planck constant is the rest energy mc^2 of the system. This hypothesis of de Broglie has thus been well confirmed by experiment. It seems surprising that this beautiful yet powerful picture of de Broglie waves is not better known. This communication is designed to help remedy the situation.

2 Stationary States

With the hindsight of Schrödinger's quantum theory, developed in the two years following de Broglie's thesis, we can associate the rest-frame oscillations assumed by de Broglie with the time dependence of the complex phase of stationary states. These are energy eigenstates ψ of the Hamiltonian H , satisfying $H\psi = E_0\psi$, which are required by the time-dependent Schrödinger equation

$$i\hbar\frac{\partial\psi}{\partial\tau} = H\psi, \quad (2)$$

to have the oscillatory time dependence

$$\psi(\mathbf{s}, \tau) = \psi(\mathbf{s}, 0) e^{-i\omega_0\tau} \quad (3)$$

with $\omega_0 = E_0/\hbar$, where \mathbf{s} indicates (possibly generalized) spatial position variables. The entire system described by ψ is seen to oscillate synchronously in phase, independent of position \mathbf{s} . Such states are the quantum analogs of normal (or natural) modes of vibration of an extended classical system. The form of the Hamiltonian H is not important here, and indeed as will be seen, de Broglie-like waves appear even with classical vibrations. The only essential property is that all parts of the system oscillate together and in phase in its rest frame. One can use the phase of the oscillation at any location to measure time, and as de Broglie suggested[2], we can think of each point in the extended system as containing a "phase clock" that is synchronized with all others throughout the system in the rest frame. The rest frame of the quantum system can be defined as the frame in which the total momentum of the system vanishes. Of course constituent parts of the system can still be interacting and in relative motion. The proper time τ of the system is the time in its rest frame and can be taken as the time displayed on the phase clocks.

There is strong experimental support for the existence of oscillations in quantum systems as described by the complex phase of the stationary-state wavefunction, but most physical measurements are based on interference phenomena

that are sensitive only to energy differences. In such cases, it is immaterial whether or not the eigenenergy E_0 includes the rest energy mc^2 or any other additive constant. However, De Broglie waves allow the additive constant to be determined.

2.1 De Broglie Waves

Special relativity tells us how to transform phenomena from any inertial frame to another. Let an active boost be applied to the system initially at rest so that after the transformation it moves with a constant net velocity \mathbf{v} in the lab. (Since only the relative motion of the system and the inertial observer is significant, the result is the same as if the system is held fixed and the observer is boosted to velocity $-\mathbf{v}$.) A position in the system before the boost can be described by a vector $s = c\tau e_0 + \mathbf{s}$ in four-dimensional spacetime, with a time component on e_0 as well as a spatial part \mathbf{s} . It is the position in the rest frame, that is, as seen by an observer at rest with respect to the system. In particular, τ is the time in the rest frame, also known as the *proper time* of the system. The position s is boosted to the spacetime position $x = ct e_0 + \mathbf{x}$ in the lab frame, given by[24]

$$x = \gamma(c\tau + \mathbf{v} \cdot \mathbf{s}/c) e_0 + \gamma(\mathbf{s}^{\parallel} + \mathbf{v}\tau) + \mathbf{s}^{\perp}, \quad (4)$$

in terms of the components of \mathbf{s} parallel (\parallel) and perpendicular (\perp) to \mathbf{v} , $\mathbf{s}^{\parallel} = \mathbf{s} \cdot \hat{\mathbf{v}} \hat{\mathbf{v}} = \mathbf{s} - \mathbf{s}^{\perp}$ with $\hat{\mathbf{v}} = \mathbf{v}/|\mathbf{v}|$. The spacetime origins of the two frames are taken to be equal: $s = 0$ at $x = 0$.

The inverse transformation is given by exchanging x and s and reversing the sign of \mathbf{v} :

$$s = \gamma(ct - \mathbf{v} \cdot \mathbf{x}/c) e_0 + \gamma(\mathbf{x}^{\parallel} - \mathbf{v}t) + \mathbf{x}^{\perp}. \quad (5)$$

In particular, the time τ in the rest frame, as recorded on the phase clocks, is the component of s on e_0 divided by c ,

$$\tau = \gamma(t - \mathbf{v} \cdot \mathbf{x}/c^2), \quad (6)$$

where t is the lab time. This part of the Lorentz transformation epitomizes what makes special relativity interesting and often nonintuitive. Under Galilean transformations, time is universal and would be the same in both rest and lab frames: $\tau = t$. In special relativity, however, time in one frame depends on both time and position variables in the other. Relation (6) can also be simply obtained from an expression of τ as the Lorentz invariant formed from the scalar product of spacetime vectors x and the proper velocity $uc = \gamma(c e_0 + \mathbf{v})$. In the notation of Refs. [16, 24] we can write

$$c\tau = \langle x\bar{u} \rangle_S = \langle (cte_0 + \mathbf{x})\gamma(e_0 - \mathbf{v}/c) \rangle_S = \gamma(ct - \mathbf{v} \cdot \mathbf{x}/c).$$

This shows the dephasing effect in the moving system: at a given instant t in the lab frame, the proper time τ in the rest frame depends on the clock position \mathbf{x}

and becomes progressively earlier at positions further in the direction of motion. Since τ is the time on the phase clocks, this shows that clocks synchronized in the rest frame become dephased or desynchronized in the lab, as illustrated in Fig. 1. What in the rest frame is a synchronized oscillation in time is seen in the lab to be a wave in space.

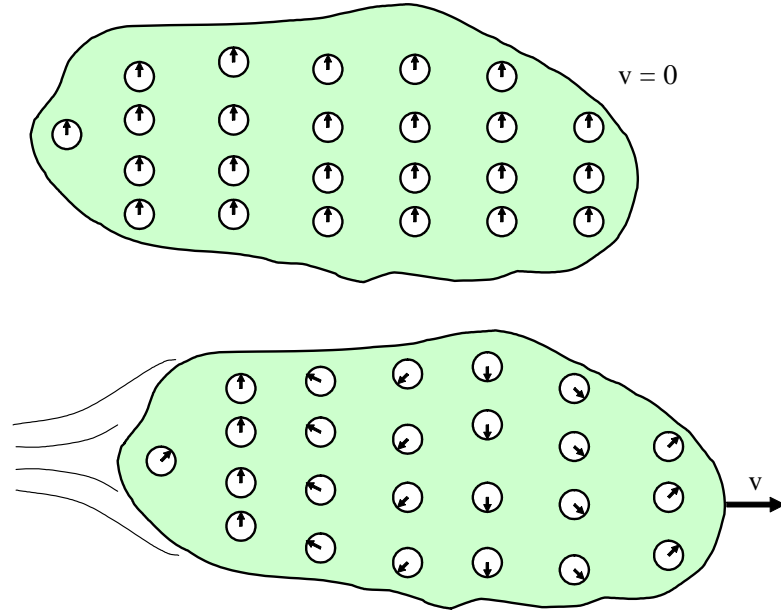


Figure 1: De Broglie waves result from the relativistic desynchronization of “phase clocks” as seen from a frame in which the clocks are moving. About one wavelength is shown for the quantum system moving with a net velocity \mathbf{v} .

The effect, we emphasize, is purely relativistic. The phase of the oscillation is the phase of the de Broglie wave by definition. In a nonrelativistic (Galilean) transformation of the velocity of the system, time would be universal and the phase clocks would remain synchronized. Any phase wave would have infinite wavelength. Under a Lorentz boost, however, time is not universal and clocks are desynchronized or dephased. The dephasing effect is distinct from time dilation. Time dilation is another relativistic effect, and it slows the rate of every moving clock by the same factor so that the period of the clock grows from τ_0 in the rest frame to $\gamma\tau_0$ when seen in the lab. However, it is not the change in clock speed but the fact that the time displayed on the moving clocks depends on their position along \mathbf{v} that gives the de Broglie wave. There is a connection between time dilation and clock dephasing: If synchronized clocks in one frame remained synchronized in the other, time dilation would not be

consistent with the relativity principle that only the relative velocity between frames matters. It is only because of clock desynchronization that observers in relative motion can both consistently hold that the other's clocks are time dilated and hence running too slow.

The stationary-state oscillations, when expressed in the spacetime coordinates of the lab, in which the system has net velocity \mathbf{v} , takes the form of a wave

$$\psi \rightarrow L\psi \left(\mathbf{x}^\perp + \gamma\mathbf{x}^\parallel - \gamma\mathbf{v}t, 0 \right) e^{-i\gamma\omega_0(t-\mathbf{v}\cdot\mathbf{x}/c^2)}. \quad (7)$$

The Lorentz operator L for the boost may mix spinor components of ψ but it is the phase $\gamma\omega_0(t - \mathbf{v} \cdot \mathbf{x}/c^2)$ that gives the wave. If the dependence of ψ on position \mathbf{s} in the rest frame is gradual compared to its oscillations, the resulting wave is approximately a plane wave of the de Broglie form. Its wavelength, the distance over which the phase changes by 2π , is seen to be $\lambda = 2\pi c^2 / (\gamma |\mathbf{v}| \omega_0)$. This agrees with the experimentally confirmed de Broglie wavelength $\lambda = h / |\mathbf{p}|$ if and only if

$$|\mathbf{p}| = \gamma m |\mathbf{v}| = \hbar \gamma |\mathbf{v}| \omega_0 / c^2,$$

that is, if and only if $mc^2 = \hbar\omega_0$ so that, as de Broglie hypothesized, the oscillations occur at what is now known as the Zitterbewegung frequency (associated with the interference of positive and negative-energy solutions of the Dirac equation).[25] The result justifies the more general relation $p = \hbar k$ between spacetime vectors. As mentioned above, the mass of a composite system includes the internal energies of interaction and motion. An alternative derivation[26] of the de Broglie relation starts by equating the group velocity of the wave with the particle velocity. This gives the first-derivative form

$$\frac{d\omega}{d|\mathbf{k}|} = |\mathbf{v}| = \frac{dE}{d|\mathbf{p}|}.$$

By putting $E = \hbar\omega$, one sees that within an additive integration constant $|\mathbf{p}| = \hbar|\mathbf{k}|$. This derivation does not fix the absolute energy and does not even require relativity. It is consistent with, but not as complete as, the requirement of special relativity applied to a synchronized oscillation in the rest frame.

It is important to recognize that the de Broglie wavelength λ is *not* that of the wave traced out by an oscillator moving at speed $|\mathbf{v}|$. For a given angular frequency ω in the lab, the oscillations would trace out one wavelength during each period $2\pi/\omega$ of oscillation, giving a wavelength $2\pi|\mathbf{v}|/\omega$ proportional to the speed. This of course also gives the relation for an arbitrary wave of wave speed $|\mathbf{v}| = \lambda\omega/(2\pi)$, and it implies that in the rest frame ($\mathbf{v} = 0$), the wavelength vanishes. However, the de Broglie wavelength becomes infinite in the rest frame. Only through the relativistic dephasing of oscillations do de Broglie waves arise, and for them, λ and hence the wave velocity is inversely proportional to the matter speed $|\mathbf{v}|$.

The physical significance of the superluminal phase velocity and its relation to clock dephasing is seen in the spacetime diagram of Fig. 2. In the rest frame, the time axis is vertical and lies along the world line of the center of momentum of the system. Ticks (cycles) of the phase clocks occur in the rest frame

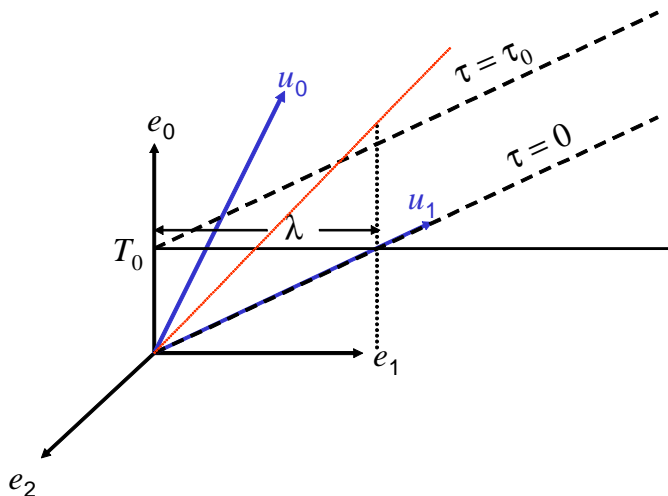


Figure 2: The hypersurface of constant phase represents an instant of time τ in the rest frame. The slope of this hypersurface on the spacetime diagram corresponds to the superluminal phase velocity in the lab. The line at 45 degrees is the light cone.

on horizontal hypersurfaces at regular intervals of $\tau_0 = 2\pi/\omega_0$ simultaneously throughout the spatial distribution of the state. In the lab frame where the system moves with a net velocity \mathbf{v} , the time axis of the rest frame becomes $u_0 = \gamma(e_0 + \mathbf{v}/c)$, which is the proper velocity in units of c . The world line of any fixed point of the rest frame is parallel u_0 , which is tilted toward the lightcone with a slope of $c/|\mathbf{v}|$. Hypersurfaces representing a given instant τ in the rest frame are tilted upward toward the lightcone by an equal angle. Thus, the slope of the rest-frame spatial axis u_1 , which lies in such a hypersurface, is $|\mathbf{v}|$. The clock phases are the same everywhere on this hypersurface. The spatial hypersurfaces corresponding to a given instant in the lab, on the other hand, are horizontal, and the phase varies periodically along the direction of \mathbf{v} on these surfaces. The constant phase on tilted hypersurfaces of fixed τ represents a plane wave propagating with superluminal wave velocity $c^2/|\mathbf{v}|$ in the lab. This constitutes the de Broglie wave.[20] The horizontal distance between successive intersections of the tilted hypersurfaces for $\tau = n\tau_0$, $n = 0, 1, 2, \dots$ with any horizontal hypersurface is the distance needed for the phase to change by 2π ; it is just the de Broglie wavelength. The period of the phase wave is the time $T_0 = \tau_0/\gamma$ given (when scaled by c) by the vertical separation of the hypersurfaces. The time component of a clock path over one cycle is the period of the clock; in the lab it has the dilated value $\gamma\tau_0$.

2.2 Conclusions

De Broglie matter waves are a natural and necessary consequence of the clock dephasing that accompanies a boost of an extended stationary state, whose oscillations are in phase in its rest frame. Hypersurfaces of constant phase are tilted in the lab frame and seen as phase waves with de Broglie wavelengths moving at superluminal velocity. Although most of the experimental evidence for the existence of stationary-state oscillations is based on interference that is sensitive only to energy or wave-vector differences, diffraction from fixed gratings confirms the absolute de Broglie wavelengths. These, in turn, require that stationary-state oscillations occur at a frequency corresponding to the total energy of the system, including its rest energy mc^2 .

References

- [1] L. de Broglie, Ph.D. Thesis, *Recherches sur la théorie des quanta*, Ann. de Phys, Ser. 10^e, t. III (1925), English transl. reprinted in *Ann. Fond. Louis de Broglie* **17**, 92 (1992).
- [2] L. de Broglie, *Non-Linear Wave Mechanics: a Causal Interpretation*, Transl. by A. J. Knodel and J. C. Miller, Elsevier, Amsterdam, 1960.
- [3] C. J. Davisson and L. H. Germer, “*The scattering of electrons by a single crystal of nickel*”, Nature **119**, 558-560 (1927).
- [4] G. P. Thomson and A. Reid, “*Diffraction of Cathode Rays by a Thin Film*”, Nature **119**, 890 (1927).
- [5] H. v. Halban, Jr. and P. Preiswerk “*Preuve expérimentale de la diffraction des neutrons*”, C. R. Acad. Sci. **203**, 73-75 (1936).
- [6] D. W. Keith, M. L. Schattenburg, H. I. Smith, and D. E. Pritchard, “*Diffraction of Atoms by a Transmission Grating*”, Phys. Rev. Lett. **61**, 1580-1583 (1988).
- [7] O. Carnal and J. Mylnek, “*Young’s Double-Slit Experiment with Atoms: A Simple Atom Interferometer*”, Phys. Rev. Lett. **66**, 2689-2692 (1991).
- [8] D. W. Keith, C. R. Ekstrom, Q. A. Turchette, and D. E. Pritchard, “*An Interferometer for Atoms*”, Phys. Rev. Lett. **66**, 2693-2696 (1991).
- [9] P. Berman, ed., *Atom Interferometry*, Academic, 1997.
- [10] P. Meystre, *Atom Optics*, Springer, Berlin 2001.
- [11] H. Pu, C. Search, W. Zhang, and P. Meystre, “*Atom Optics - From de Broglie Waves to Heisenberg Ferromagnets*”, Fortschr. Physik **50**, 664-669 (2002).

- [12] M. Arndt, O. Nairz, J. Voss-Andreae, C. Keller, G. van der Zouw, and A. Zeilinger, “*Wave-particle duality of C60*”, *Nature* **401**, 680–682 (1999).
- [13] B. Brezger, L. Hackermüller, S. Uttenthaler, J. Petschinka, M. Arndt, and A. Zeilinger, “*Matter-Wave Interferometer for Large Molecules*”, *Phys. Rev. Lett.* **88**, 100404 (2002).
- [14] M. Köhl, T. W. Hänsch, T. Esslinger, “*Measuring the Temporal Coherence of an Atom Laser Beam*”, *Phys. Rev. Lett.* **87**, 160404 (2001).
- [15] L. de Broglie, “*Ondes et Quanta*”, *C. R. Acad. Sci.* **177**, 507-510 (1923).
- [16] W. E. Baylis, *Electrodynamics: a Modern Geometric Approach*, Birkhäuser, Boston, 1999.
- [17] E. Schrödinger, *Abhandlungen zur Wellenmechanik*, J. A. Barth, Leipzig, 1926 (1st ed.) and 1928 (2nd ed.), transl. by J. F. Shearer and W. M. Deans in *Collected Papers on Wave Mechanics*, Blackie & Son, London, 1928.
- [18] R. P. Feynman, “Space-Time Approach to Non-Relativistic Quantum Mechanics”, *Rev. Mod. Phys.* **20**, 367–387 (1948).
- [19] R. P. Feynman, *QED: the Strange Theory of Light and Matter*, Princeton Univ., Princeton, 1985
- [20] L. de Broglie, “*Interpretation of Quantum Mechanics by the Double Solution Theory*” (English translation by M. Surdin of paper in *Foundations of Quantum Mechanics - Rendiconti della Scuola Internazionale di Fisica Enrico Fermi*, Il Corso, B. d’Espagnat ed., Academic Press, N.Y.1972), *Ann. Fond. Louis de Broglie* **12**, 1–23 (1987).
- [21] L. de Broglie, *The Current Interpretation of Wave Mechanics*, Elsevier, Amsterdam, 1964.
- [22] D. Bohm and B. J. Hiley, *The Undivided Universe: An Ontological Interpretation of Quantum Theory*, Routledge, London, 1993.
- [23] P. R. Holland, *The Quantum Theory of Motion*, Cambridge Univ., Cambridge, 1993.
- [24] W. E. Baylis, “*Relativity in Introductory Physics*”, *Can. J. Phys.* **82** (11), 853—873 (2004).
- [25] P. Strange, *Relativistic Quantum Mechanics*, Cambridge Univ., Cambridge, 1998.
- [26] A. Messiah, *Quantum Mechanics*, vol. I, North Holland, Amsterdam, 1961, p. 52.