GEOGEBRA-NA 2014
Proceedings of the Fifth North American GeoGebra Conference:
Explorative Learning with Technology

Editors:
Dragana Martinovic, University of Windsor
Zekeriya Karadag, Bayburt University
Doug McDougall, University of Toronto
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History of GeoGebra-NA

One of the results stemming from the First International GeoGebra conference held on July 14-15, 2009, in Linz, Austria was in organizational activities around planning regional conferences in Spain, Turkey, Argentina, South America, and Norway.

In an attempt to engage the communities of mathematicians, mathematics educators, and software developers in discussions around the potential of technology for learning and teaching of mathematics, the First North-American GeoGebra conference (GeoGebra-NA2010), held on July 28-29, 2010 in Ithaca, NY, laid the foundations for a series of conferences in North America and consequently to this conference in Canada. The idea is to hold annual conferences interchangeably in the US, Canada and Mexico.

Goals of GeoGebra-NA

The global mission of this conference is to build a community of mathematicians, mathematics educators, and classroom teachers who can benefit from the potential of Dynamic Interactive Mathematics Learning Environments (DIMLE), and in particular GeoGebra. The dynamism and interactivity of the DIMLE environments make explorative learning possible by allowing for the seemingly continual change of mathematical objects, as well as for the seamless inter-dependence between different mathematics representations.

With this joint effort, we hope to transform mathematics instruction and curricula through creative development of resources, practical experimentation with the software, and research on the related learning and teaching gains.

The intellectual merit of GeoGebra-NA2014 and its scientific focus is in striving to achieve the following objectives:

1. Identifying an agenda of critical research and development needs in the field; exploring collectively questions around research processes in relation to GeoGebra and other similar software.

2. Creating synergies between developers, mathematicians, educational researchers and teachers-practitioners which will influence research on how technology supports education.

3. Exploring ways to reach and engage diverse communities in promotion of mathematics; producing a freely available repository of quality instructional methods and materials for integrating GeoGebra into mathematics education.
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- Kitty Yan, OISE, University of Toronto, ON
- Sarah Bennett, OISE, University of Toronto, ON
Welcome from the Chairs:

Welcome to the Fifth North-American GeoGebra conference and the second GeoGebra conference held in Canada. We are excited with the prospect of this conference to build, support and sustain a community of mathematicians, mathematics educators, classroom teachers, graduate students and software developers who can benefit from the potential of computer applications in mathematics education.

We have received a number of proposals for the papers and the working groups. Nine papers, two working groups, and two workshop descriptions found place in these proceedings.

We are thankful to our supporters: The Fields Institute for Research in Mathematical Sciences, Ontario Institute for Studies in Education, and the GeoGebra Institute of Canada. We are also pleased to announce that GeoGebra-NA 2015 will be held in Mexico and hope that we will all meet there again!

Dragana, Doug, and Zekeriya
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Zekeriya Karadag
PLENARY SESSIONS

Plenary Speakers (Short Bios)

**Dr. Milner-Bolotin** is an Assistant Professor in Science Education at the Department of Curriculum and Pedagogy at the University of British Columbia in Vancouver, Canada. She is actively engaged with BC Association of Physics Teachers, as well as with the American Association of Physics Teachers. In addition, she is an adjunct professor at the Department of Physics at Ryerson University in Toronto, Canada. Her research comprises a study on how modern technology-based teaching methods (such as: interactive lecture experiments based on the live data collection (Logger Pro-Vernier); electronic feedback systems (clickers); and tablet PCs-based collaborative teaching methods and computer simulations) affect: student academic achievement and interest in science; classroom environment as expressed in the nature of student-teacher and student-student interactions; performance of traditionally underrepresented groups in sciences and engineering (i.e. girls, minority students); science teachers’ content and pedagogical knowledge and their motivation in improving their teaching and their students’ learning. Blog: http://blogs.ubc.ca/mmilner/

**Giancarlo Brotto** is the SMART Technologies™ Education Consultant for Ontario. He works in partnership with technology decision makers, administrators and educators to ensure that their adoption of SMART™ products leads to effective integration, sustained use, and maximum impact on teaching and learning. Before joining SMART™ Giancarlo was an Exemplary Educator and taught mathematics for six years in a high school laptop program in Ontario. He also spent several years working in the Statistics Department at the University of Toronto while completing his undergraduate specialist degree in mathematics and Bachelor of Education degree. Although his current role finds him away from the mathematics classroom, his passion continues to reside in helping transform students’ perception of learning and understanding mathematics.

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CLOSING THE RESEARCH-PRACTICE GAP THROUGH INNOVATIVE TECHNOLOGY USE
IN STEM TEACHER EDUCATION

Marina Milner-Bolotin
The University of British Columbia

Abstract
Over the last half century, there have been ample research highlighting the value of active engagement in Science, Technology, Engineering and Mathematics (STEM) education (Bransford, Brown, & Cocking, 2002; Hake, 1998; Kalman, 2008). There also has been a significant progress in the technological tools available to modern STEM educators to promote research-based pedagogies, such as GeoGebra, PhET simulations, various probeware, etc. Yet the gap between what we know about how students learn STEM and how we teach these disciplines seems to remain largely intact. In this paper we present our recent research on the use of educational technologies in mathematics and physics teacher education and shed light on what we know about the effects of these technologies on the development of teacher-candidates’ Pedagogical Content Knowledge (Shulman, 1986) and their willingness and ability to implement active engagement pedagogies during their practicum experiences and possibly after graduation. We focus specifically on the role of technologies in three aspects of STEM teacher-education: helping teacher-candidates develop (a) Pedagogical Content Knowledge, (b) positive attitudes about the use of technologies in STEM teaching and learning, and (c) the capacity and willingness to explore novel applications of technologies that promote active student engagement with STEM disciplines.

Keywords: STEM teacher preparation, educational technologies, PeerWise, Peer Instruction, Logger Pro, GeoGebra

Over the last half century, there have been ample research highlighting the value of active engagement in Science, Technology, Engineering and Mathematics (STEM) education (Bransford et al., 2002; Hake, 1998; Kalman, 2008). There also has been a significant progress in the technological tools available to modern STEM educators to promote research-based pedagogies, such as dynamic software (i.e., GeoGebra), PhET simulations, various probeware (i.e., Logger Pro probeware), and online interactive problem solving tutorial systems (i.e., WebAssign). Yet the gap between what we know about how students learn STEM and how contemporary educators teach these disciplines seems to remain largely intact. In this paper we present and analyze recent research on the use of educational technologies in mathematics and physics teacher education and shed light on what we know about the effects of these technologies on the development of teacher-candidates’ Pedagogical Content Knowledge (Shulman, 1986) and their willingness and...
ability to implement active engagement pedagogies during their practicum experiences and after graduation.

The research on educational technologies, their affordances and limitations, technology-enhanced educational practices, and technology-enhanced professional development for teachers, has grown significantly in the last decades (Jonassen & Land, 2012; Jonassen, Campbell, & Davidson, 1994; Luft & Hewson, 2014; Russel & Martin, 2014; van Driel, Berry, & Meirink, 2014). In this paper, we focus specifically on the role of technologies in Science, Technology, Engineering and Mathematics (STEM) teacher-education. Through the action research study (Kemmis, 2011) in the context of the Physics Methods course at the Teacher Education Program at the University of British Columbia, we attempt to expand our understanding of how modern educational technologies can help teacher-candidates develop (a) Pedagogical Content Knowledge (Shulman, 1986), (b) positive attitudes about the use of technologies in STEM teaching and learning, and (c) the capacity and willingness to explore novel applications of technologies that promote active student engagement with STEM disciplines. To answer these questions, we begin by outlining the theoretical lens through which I will analyze and present the results of our research.

**Technology-Empowered Deliberate Pedagogical Thinking**

At the dawn of computer-enhanced STEM education, Alan Key - one of the pioneers of the field - gave an interview on the role of computers in education (Kay, 1987). In that interview he raised a concern that if we are not deliberate about why we use educational technologies and how we want to use them in order to support student learning, we will allow “the educational technology tail to wag the pedagogical dog”. Unfortunately this is what often happens in our classrooms - educational technologies are acquired (and too often stowed away in the classroom cabinets), yet the pedagogical practices remain intact: this prompted another noted scholar, Prof. Larry Cuban from Stanford University, to lament that computers in our classroom are often “oversold and underused” (Cuban, 2001). This was true largely true then and unfortunately if we expand the definition of “computers” to include a wide range of contemporary technologies, it remains true today: the presence of new technologies in the classroom does not ensure that that teachers are ready to embrace pedagogically effective technology-enhanced educational practices. The latter requires technology-empowered deliberate pedagogical thinking. This concept emphasizes that technology should empower teachers to achieve their pedagogical goals and not to be the end-goal within itself. In the context of STEM education, technology-empowered deliberate pedagogical thinking underscores the pedagogical considerations that view technology as a means to an end of promoting active and meaningful student engagement with STEM and inspiring
students to pursue STEM-related fields in the future (Deslauriers, Schelew, & Wieman, 2011; Let's Talk Science, 2013; Wieman, 2012).

The concept of technology-empowered deliberate pedagogical thinking is especially relevant to STEM teacher education. In order for teachers to think pedagogically about the use of technology in their classrooms, they have to have extensive content and pedagogical knowledge of the subjects they are teaching. In his seminal 1986 American Educational Research Association (AERA) presidential address on the growth of knowledge in teaching, the former AERA president, Lee S. Shulman (1986), challenged us to re-think how we prepare our teachers and what we expect of them to know in terms of their knowledge of the subject matter they are teaching (Content Knowledge, often referred to as CK) and of the pedagogical approaches and competences in the teaching of their subject (Pedagogical Knowledge or PK). Shulman pointed out that in 1980s many American teacher education programs suffered from the “missing paradigm” problem, as in their attempt to incorporate the results of the generic educational research into teacher education they often neglected the nature of the context – the subjects teachers were supposed to be prepared to teach:

My colleagues and I refer to the absence of focus on subject matter among the various research paradigms for the study of teaching as the “missing paradigm” problem. The consequences of this missing paradigm are serious, both for policy and for research... Research programs that arose in response to the dominance of process-product work accepted its definition of the problem and continued to treat teaching more or less generically or at least as if the content of instruction were relatively unimportant (Shulman, 1986, p. 6) (italics are added).

The “missing paradigm problem” is also relevant to many teacher education programs in Canada, where subject-specific methods courses are given very limited attention, while teacher-candidates spend most of their time taking general education courses. This leaves them to struggle alone with the hardest educational task – transferring general educational theoretical knowledge into subject-specific pedagogical practice (Milner-Bolotin, 2014b). This has serious negative implications on how teacher-candidates implement educational technologies in their teaching during the practicum and after graduation: as technology-empowered deliberate pedagogical practice requires teachers not only to possess significant Pedagogical Content Knowledge (PCK) but also to have the technology and subject-specific knowledge. The latter is often referred to as Technological—Pedagogical and Content Knowledge or TPCK (Figure 1) (Milner-Bolotin, Fisher, & MacDonald, 2013; Mishra & Koehler, 2007; Mishra, Koehler, & Henriksen, 2011). This paper describes the study investigating research-based practices aimed at helping future mathematics and physics teachers develop TPCK necessary for promoting technology-empowered deliberate pedagogical thinking in their classrooms during the school practicum and beyond. First, we focus on research-based
technology-enhanced pedagogies that have a significant potential for helping teacher-candidates develop necessary TPCK. Second, we consider how we might be able to help teacher-candidates develop positive attitudes about the role of technology in STEM education during their subject-specific methods courses. Third, we examine if and how a technology-enhanced pedagogical approach implemented in the semester long Physics Methods courses can support teacher-candidates in developing the capacity for technology-empowered deliberate pedagogical thinking.

Figure 1. TPCK Framework by Mishra and Koehler (2007).

Making a Case for Technology-Enhanced Pedagogies in STEM Teacher Education

As discussed earlier, pedagogically successful implementation of technology-enhanced pedagogies in STEM classrooms depends largely on instructor’s TPCK (pedagogical decisions) and on the availability of technological tools that can facilitate student active and meaningful engagement. In this study, a number of educational technologies were used: the Peer Instruction pedagogy (Mazur, 1997) was paired with the online collaborative tool PeerWise (Denny, 2014; Milner-Bolotin, 2014c); live data collection and analysis using Logger Pro probeware (Vernier-Technology, 2014); computer simulations (Perkins et al., 2006) and dynamic mathematical software (Hohenwarter, 2014) were implemented in the Physics Methods course. These technologies were used to support physics teacher-candidates in the development of deliberate pedagogical thinking, positive attitudes about educational technologies and the capacity for engaging with new technology-enhanced pedagogies.
Peer Instruction in STEM Teacher Education

Peer Instruction is a pedagogical approach based on Conceptual Change Theory (Jonassen & Easter, 2012; Mazur, 1997; Özdemir & Clark, 2007). It emphasizes the value of student prior knowledge and focuses on eliciting and confronting student misconceptions through student collaboration and peer learning. Peer Instruction utilizes Classroom Response Systems (clickers) to engage students in responding to multiple-choice conceptual questions (Milner-Bolotin, 2004). Its successful implementation relies on high-quality multiple-choice questions that target core science concepts while using common misconceptions as distractors (Milner-Bolotin, 2004, 2014a; Milner-Bolotin, Antimirova, & Petrov, 2010).

Peer Instruction has proven to be pedagogically effective in STEM classrooms at the undergraduate (Hake, 1998; Kalman, Milner-Bolotin, & Antimirova, 2010; Lasry, Mazur, & Watkins, 2008) and secondary levels (Fagen, Crouch, & Mazur, 2002). However, it is yet to become commonly used in STEM teacher education (Cha, 2013). One explanation for this phenomenon might be the perception that Peer Instruction is only beneficial for large classes. Yet as we shall see later, this is not the case: this pedagogy is also effective for small groups where students (who often know each other well) might feel threatened and embarrassed to exhibit the lack of understanding. In the version of Peer Instruction implemented in the Physics Methods courses described here (Figure 2), teacher-candidates use clickers to submit their individual responses to a multiple-choice conceptual question posed by the instructor. While instructor can check the responses of individual students after the end of class, during the class all students’ responses are anonymous and they are not required to reveal their individual vote to the entire class. Then the histogram of their responses is shared with the class without revealing the correct answer. It is followed by small group student discussions.

When students struggle with answering the question correctly, as shown in Figure 2, the instructor might ask additional questions or invite volunteers to explain the reasoning behind each possible answer. This is especially valuable for teacher-candidates who might not be aware of their own lack of understanding, common misconceptions and conceptual difficulties. After peer discussion, students vote individually on the same question. When the second histogram is shown, the students can see whether group thinking shifted as a result of the discussion. Since the shift can be either in the right or wrong direction, the instructor’s responsibility is to facilitate the discussion to help students uncover their own difficulties and discover accurate scientific understanding (Lasry et al., 2014; Lasry, Watkins, Mazur, & Ibrahim, 2013). Therefore, the Peer Instruction process ends with a teacher-facilitated discussion summarizing major concepts while exploring alternative explanations and the reasoning behind each possible response.
Figure 2. Example of Peer Instruction pedagogy (6-8 minutes per question). In this example, the physics teacher-candidates were initially confused (only four chose the correct answer [D]). Yet, after the group discussion, 10 out of 13 were able to answer the problem correctly (the remaining 3 prospective teachers did not vote the second time).

Pedagogically effective conceptual questions avoid extensive calculations, yet elicit and challenge students' core conceptual understanding. After committing to a particular answer, students are challenged to examine their own reasoning while focusing on the why instead of what.
questions. This examination of basic and complex concepts helps students develop scientific ways of thinking and acquire reasoning skills. It is especially valuable for future teachers. In a traditional STEM classroom, conceptual difficulties can be easily hidden and remain largely unchallenged. Extensive research collected over the last 25 years indicates that Peer Instruction classrooms promote the development of deep conceptual understanding through active student engagement (Hake, 1998; Lasry, 2008; Lasry et al., 2008).

Supporting STEM teacher-candidates in developing foundations of TPCK is one of the central goals of teacher education. Yet, our experience with physics teacher-candidates indicates that they often lack the Content Knowledge, relevant to the secondary physics curriculum, which extends beyond factual memorization and is vital for building a strong PCK foundation (Milner-Bolotin et al., 2013; Nashon & Anderson, 2004). This prevents teacher-candidates from developing the Pedagogical Knowledge and consequently from implementing research-based technology-enhanced pedagogical practices in their own classroom during the practicum. This experience informed our decision to make Peer Instruction a significant component of our Physics Methods course: we wanted teacher-candidates to experience this pedagogy both as students and as prospective teachers (Milner-Bolotin et al., 2013). However, to open them up to the possibilities of this technology-enhanced active engagement pedagogy, we needed to empower teacher-candidates in designing and evaluating conceptual multiple-choice STEM questions. This was accomplished through a two-step process. First, we incorporated an online database of conceptual multiple-choice questions developed by our team into the course activities (Milner-Bolotin, 2014a). Second, we incorporated the PeerWise online tool to support teacher-candidates' collaboration on designing, evaluating and improving conceptual questions of their own (Milner-Bolotin, 2014c).

**PeerWise in STEM Teacher Education**

The implementation of Peer Instruction in the Physics Methods course in the first year of the study made it clear that the key to its success is not the technology, but teacher-candidates’ ability to recognize and even design powerful conceptual questions that challenge students to apply physics concepts in novel contexts (Beatty et al., 2008; Beatty, Gerace, Leonard, & Dufresne, 2006; Milner-Bolotin et al., 2013). The question design process improves significantly when teacher-candidates can collaborate and receive feedback on their questions. This allows teacher-candidates to not only refine their own questions but also to learn how to provide constructive feedback and evaluate the quality of multiple-choice questions designed by their peers. PeerWise is a free online tool that provides these opportunities through enabling users to author, share, evaluate, rate, and discuss multiple-choice questions (Denny, 2014). It allows students (in this case,
teacher-candidates) and course instructors to actively participate in the collaborative design of a shared database of multiple-choice questions (Milner-Bolotin, 2014c). The course instructor can participate in the discussions, as well as view detailed statistics about student participation, including which questions are answered most frequently, how often a user authors or answers a question, and the quality of each student’s work as evaluated by their peers. At the end of the course, all class members can benefit from having an access to a collection of collaboratively designed multiple-choice conceptual questions created and vetted by their peers.

Modeling Software and Data Collection Technology in STEM Teacher Education

Peer Instruction and PeerWise can support the development of conceptual thinking and PCK of STEM teacher-candidates. However, STEM teachers should also have an opportunity to use modern technologies such as data collection technology (Vernier-Technology, 2014), computer simulations (Perkins et al., 2006) and dynamic mathematic software (Hohenwarter, Hohenwarter, & Lavicza, 2008) to learn how to use real or virtual data to help students develop scientific reasoning skills. For example, Finkelstein and his colleagues have shown that allowing students to do a virtual lab prior to performing a real life can significantly promote their understanding of the science concepts involved (Finkelstein et al., 2005). Moreover, computer simulations allow students to manipulate variables and build conceptual connections that are difficult to do otherwise. Figure 3 shows a screen shot of a PhET computer simulation called Calculus Grapher (http://phet.colorado.edu/en/simulation/calculus-grapher).

Figure 3. Calculus Grapher allows students to connect the concepts of derivative and integral of a function and visualize them.
In our own research we have shown the value of using real time data collection and analysis technology, such as Logger Pro (Vernier-Technology, 2014), both during lectures and as part of the assessment of student understanding (Milner-Bolotin, 2012; Milner-Bolotin, Kotlicki, & Rieger, 2007; Milner-Bolotin & Moll, 2008). For example, one of the concepts that is often difficult for the students to understand is the concept of apparent weight (Milner-Bolotin, 2008). To help students understand it we collected real time values of apparent weight (a normal force exerted by the jar on the scale) of a jar of water placed on a scale inside the moving elevator. The collected data helped students understand the concept and link the physics learned in the classroom to their own everyday life experiences (Figure 4).

Figure 4. A water jar was placed on a Logger Pro force plate (digital scale) inside a moving elevator in order to record apparent weight values during the ride: weight and apparent weight problem.

While computer simulations provide an environment where students can manipulate a limited number of parameters, dynamic mathematical software, such as GeoGebra (Hohenwarter, 2014) provides students with the tools for open exploration. It has been shown that using dynamic mathematics software allows students to grapple with authentic mathematical problems that they could not have done in the past (Hohenwarter et al., 2008; Hohenwarter, 2014; Sherman, 2010). Moreover, in order to use GeoGebra, the students have to formulate and express their mathematical thinking through geometrical construction as opposed to free drawing: they have to uncover the mathematical rules and relationships for specific constructions. As a result, GeoGebra allows students to experience construction in geometry and explore the interdependencies and invariant properties of mathematical objects. If the construction was done properly, these invariant properties should remain intact under the dynamic manipulation. For example, if the students discovered a proper construction strategy for inscribing a circle in a triangle (Figure 5), modifying the triangle should modify the inscribed circle, but the circle should remain inscribed. GeoGebra allows students to experience the geometrical invariance and discover the relationships that would have remained hidden (or memorized) in a traditional instruction. GeoGebra also prompts students...
to ask and answer What-If questions that could not have been answered empirically using the tools of the traditional classroom. Thus, the process of “questioning the obvious” through dynamic software opens doors to unexpected and pedagogically invaluable “aha-moments”. Since the software is free, its use is not limited to the physical classroom and the students can conduct their explorations at home. This is especially useful for the students participating in online mathematics courses (http://audrey-mcsquared.blogspot.ca/). Most importantly, dynamic mathematical software promotes divergent thinking inviting students to suggest multiple solutions to a problem instead of memorizing the solution suggested by the teacher.

Figure 5. A screen shot of teacher-candidate’s exploration of the strategy for inscribing a circle inside a triangle that will work for all kinds of triangles – acute, obtuse and right triangles.

In the following section we discuss the context of the study.

Study Context

This ongoing study spans the period of three years (2012-2015) and it takes place at the Teacher education Program at the University of British Columbia. In each year one Physics Methods course is offered. The Physics Methods course described in this study typically has an enrollment of 8-14 secondary physics teachers-candidates. In order to be admitted into a year-long Secondary Teacher Education Program in the province of British Columbia, teacher-candidates must earn a Bachelor of Science degree or its equivalent, or be concurrently enrolled in a respected undergraduate STEM program at the same institution. During the first year of the study, 13 teacher-candidates were enrolled in the Physics Methods course, while in the second year of the study, the ten teacher-candidates enrolled in the course, and in the third year the enrollment decreased to 8 students. Physics teacher-candidates often pursue a dual certification, such as physics and mathematics, physics and chemistry, or physics and another science subject. Their academic backgrounds also vary from a traditional B.Sc. in physics degree, to an Engineering degree or a M. Sc. or even a Ph.D. degree in relevant subject. Some of them had prior teaching experience outside of Canada, but none had taught in Canadian secondary schools. All of the teacher-candidates were preparing to teach physics as their primary subject (Table 1).
The Physics Methods course in all these years was led by an instructor (the author) with over 20 years of experience in physics teaching and physics education research. The instructor was assisted by a Graduate Teaching Assistant with a B.Sc., pursuing graduate studies in physics education research. The course (three credits) ran for one 13-week term during the first semester of the Teacher Education Program (66 credits). It included two weekly, 90 minute meetings (39 in-class hours in total). In the middle of the course, teacher-candidates participated in a short two-week school practicum to get acquainted with a local school where they will be teaching for 10 weeks (long practicum) during the following semester. Six weeks after completing the physics methods course teacher-candidates begin their long school practicum, consisting of ten weeks of intensive teaching in a public secondary school. It follows by three weeks of extended practicum which offers an opportunity to explore informal learning environments such as science museums, aquarium, Let’s Talk Science, and other science-related organizations.

Table 1. An example of a typical demographics of teacher-candidates in the Physics Methods course discussed in this paper (Year 2 of the study): the data was collected during the second year of the study.

<table>
<thead>
<tr>
<th>No.</th>
<th>Location</th>
<th>Degree(s)</th>
<th>Teachable Subjects</th>
<th>Program Type</th>
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<tr>
<td>1</td>
<td>In province</td>
<td>B.Sc. Physics</td>
<td>Physics</td>
<td>Concurrent B.Ed.</td>
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<tr>
<td>2</td>
<td>In province</td>
<td>B.Sc. Physics</td>
<td>Physics</td>
<td>Concurrent B.Ed.</td>
</tr>
<tr>
<td>3</td>
<td>In province</td>
<td>B.Sc. Astronomy</td>
<td>Physics</td>
<td>B.Ed.</td>
</tr>
<tr>
<td>4</td>
<td>In province</td>
<td>B.A.Sc. Applied Physics MBA</td>
<td>Physics, Business, ESL</td>
<td>B.Ed.</td>
</tr>
<tr>
<td>5</td>
<td>In province</td>
<td>B.A.Sc. Mechanical Engineering MBA</td>
<td>Physics</td>
<td>B.Ed.</td>
</tr>
<tr>
<td>6</td>
<td>International</td>
<td>B.Sc. (Physics, Math, Chemistry)</td>
<td>Physics, Math, IB</td>
<td>B.Ed.</td>
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<tr>
<td>7</td>
<td>In province</td>
<td>B.Sc. (Hons.) Physics M.Sc.</td>
<td>Math and Physics</td>
<td>B.Ed.</td>
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<tr>
<td>8</td>
<td>In province</td>
<td>B.Sc. Physics</td>
<td>Physics, Science, Math</td>
<td>B.Ed.</td>
</tr>
<tr>
<td>9</td>
<td>In province</td>
<td>B.Sc. Physics and Math</td>
<td>Physics and Math</td>
<td>B.Ed.</td>
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<td>10</td>
<td>In province</td>
<td>B.Sc. Physics, Math minor</td>
<td>Physics, Math</td>
<td>B.Ed.</td>
</tr>
</tbody>
</table>

Four summative assessments and a holistic in-class participation grade were used to determine a Pass or Fail grade for this Methods course: teacher-candidates have to earn a grade of 80% of higher to pass the course (Figure 6).
Figure 6. Summary of course assignments.

The course is cyclical in nature, with challenges and successes in assignments outside of class informing the use of in-class time and vice versa. Peer Instruction is used to model questioning skills throughout. Questions submitted on PeerWise are incorporated into class time as a foundation for discussions about content and pedagogy. Every teacher-candidate is required to submit 3 conceptual questions every week and to comment and respond to at least five conceptual questions submitted by peers. These online discussions inform teacher-candidates’ development of PeerWise questions and help course instructor uncover topics that require additional discussion during class time.

The majority of in-class hours are spent answering and discussing conceptual questions from multiple perspectives. In addition, teacher-candidates are introduced to relevant technologies through lab activities, demonstrations, and interactive exploration of resources such as online simulations and probeware. Figure 7 summarizes the overall course structure including the time spent in class with teacher-candidates.
Figure 7. Summary of in-class activities.

For this study the data was collected through classroom observations, the analysis of conceptual questions designed by teacher-candidates and submitted through PeerWise (in total more than 450 conceptual multiple choice questions was submitted by teacher-candidates in each year of the study), pre-practicum and post-practicum interviews with the teacher-candidates, focus groups and teacher-candidates’ feedback during and after the end of the course (via anonymous course surveys).

**Results and Discussion**

This study aimed at expanding our understanding of how modern educational technologies can help teacher-candidates develop technology-empowered deliberate pedagogical thinking. As we discussed earlier, this thinking is founded on three pillars (a) Pedagogical Content Knowledge (Shulman, 1986), (b) positive attitudes about the use of technologies in STEM teaching and learning, and (c) the capacity and willingness to explore novel applications of technologies that promote active student engagement with STEM disciplines. The results presented below address each one of these pedagogical pillars.

**Educational Technology as a Tool for Promoting Teacher-Candidates’ PCK**

In order to evaluate how technology-enhanced pedagogies used in the Methods course influenced teacher-candidates’ PCK, we analyzed the quality of conceptual multiple-choice questions submitted by teacher-candidates using a special rubric we designed for the study (Milner-
Bolotin et al., 2013) (Table 2). This Rubric was based on Bloom’s Taxonomy of Educational Objectives in Cognitive Domain (Bloom, 1956) and included both Pedagogical Knowledge and Content Knowledge dimensions from the PCK framework. The Rubric consisted of Likert-type scales evaluating the pedagogical quality of the questions (accuracy, clarity, focus on specific conceptual difficulties), including the quality of the distractors, the quality of the explanations, the use of multiple representations and the ability to relate the question to students’ lives. In order to produce reliable results, three independent researchers evaluated teacher candidates’ conceptual questions and all the discrepancies in rating were discussed and reconciled. The final Cronbach alpha for the study exceeded 0.75 for each one of the three years.

Table 2. A rubric for evaluating pedagogical and content effectiveness of conceptual questions (CA – Correct Answer and IA – Incorrect Answer)

<table>
<thead>
<tr>
<th>Message Level</th>
<th>Cognitive level</th>
<th>Targeting student difficulties</th>
<th>Science accuracy</th>
<th>Distractors’ quality</th>
<th>Answer justification</th>
<th>Question clarity</th>
<th>Multiple representation &amp; MRs</th>
<th>Part of a sequence</th>
<th>Originality</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Knowledge</td>
<td>1 Knowledge</td>
<td>Doesn’t target any concept. difficulty</td>
<td>Major mistakes in the question &amp; in solutions</td>
<td>All irrelevant distractors</td>
<td>No answer justification</td>
<td>Unclear, misleading &amp; inaccurate</td>
<td>1 MR</td>
<td>No</td>
<td>Exactly copied from a known source</td>
</tr>
<tr>
<td>Comprehension</td>
<td>2 Comprehension</td>
<td>Targets a minor concept ineffectively</td>
<td>An accurate question but an inaccurate &amp; unclear solution</td>
<td>Incomplete CA justification; no IA justification</td>
<td></td>
<td></td>
<td>2 MRs</td>
<td>Yes</td>
<td>Copied with minor modifications</td>
</tr>
<tr>
<td>Application</td>
<td>3 Application</td>
<td>Targets a minor concept effectively</td>
<td>The question is clear; the solution is accurate but unclear</td>
<td>Half of distractors are meaningful</td>
<td>Incomplete CA &amp; IA justification</td>
<td>Minor problems in the question or in the solution</td>
<td>3 MRs</td>
<td></td>
<td>Copied with some modifications</td>
</tr>
<tr>
<td>Analysis</td>
<td>4 Analysis</td>
<td>Targets a few conceptual difficulties</td>
<td>Both question &amp; solution are somewhat clear &amp; accurate</td>
<td>Complete and accurate CA justification only</td>
<td></td>
<td></td>
<td>4 MRs</td>
<td></td>
<td>Copied with interesting modifications</td>
</tr>
<tr>
<td>Synthesis/evaluation</td>
<td>5 Synthesis/evaluation</td>
<td>Clearly targets major conceptual difficulties</td>
<td>Both question &amp; solution are clear &amp; accurate</td>
<td>All distractors are meaningful</td>
<td>Complete &amp; accurate CA &amp; IA justification</td>
<td>Both quest. &amp; solution are very clear &amp; accurate</td>
<td>5 MRs</td>
<td></td>
<td>Original question</td>
</tr>
</tbody>
</table>
The results of that analyses during each one of the three years of the study have shown significant improvement in the quality of teacher-candidates’ multiple-choice conceptual questions over the term of the course (Milner-Bolotin, 2014c). The analysis also shows that as the course progressed, teacher-candidates were able to incorporate technology into the questions, themselves. For example, they often used real-life data and computer simulations in their questions in. In the last two years of the study, we also collected teacher-candidates’ comments and discussions related to the design of the questions which they posted on PeerWise. These discussions clearly indicated the growth in teacher-candidates’ PCK: by coming up with their own conceptual multiple-choice questions and solving the questions designed by their peers, teacher-candidates were able to deepen their own PCK. In the third year of the study, they were also required to provide feedback on the conceptual questions designed by others. In order to provide meaningful feedback to each other they often had to rely on relevant educational-research literature and expertise of STEM educators around the world.

The content and pedagogically-driven discussions emerged on PeerWise often extended into the classroom and helped generate new understanding of the role of conceptual understanding and research-based pedagogical strategies in STEM teaching, prompting teacher-candidates to reflect on its value in their own practice. This is an excerpt from the interview emphasizing the role of conceptual understanding in STEM:

> ...physics is...not about applying formulas, and doing math. It is...about gaining an appreciation of the world around us. And, being able to use your understanding and extrapolate...explain what’s happening around you. [It] has nothing to do with math formulas. (Teacher-candidate A)

The emphasis on meaningful learning and conceptual understanding helped teacher-candidates to re-evaluate the role of technology in STEM education. This is discussed in the following section.

**Educational Technology as a Tool for Promoting Positive Attitudes about Technology**

We also collected extensive evidence that modeling technology-empowered deliberate pedagogical thinking in the methods courses through the use of technologies helped teacher-candidates acquire positive attitudes about the role of educational technologies in STEM learning.

The following are the excerpts from the interviews with teacher-candidates:

> It (clickers) really opens the door for... discussions between people... regarding a) ... what is the right answer, and b) how would you explain that to ... either teacher-candidates or to your potential students? (Teacher-candidate B)

Another teacher-candidate explained how they viewed the role of technology in the STEM classroom:
So, if you set it up in a dynamic where... different types of people have [different needs], so if you need to talk to someone, you still get that; if you need silence, you get to think on it on your own, and then people aren’t so stressed... And they actually get to argue and talk back and forth and they’ll remember it more. So for them, I think they’ll master it more.
(Teacher-candidate C)

Many of teacher-candidates emphasized the role of technology in promoting student active engagement:

“... some of the physics 11s who are just doing it to do a science, and are just, ‘Alright, Physics, I’ll try it out.’ Some of them were not as engaged, and I think doing the... voting-style questions helped get them more into it and more involved. So I’d say... it’s helpful to get those students who hide at the back in these 30 person classes.” (Teacher-candidate D)

However, one of the most interesting findings of the study is the extent to which teacher-candidates internalized the values of technology-empowered deliberate pedagogical thinking: technology is a tool that helps promote deliberate research-informed pedagogies:

It wasn’t just the clickers alone. It was also in... the presentation of the question. It wasn’t a simple plug in the answer-type question. It had to be conceptual, in which you could..., the Bloom’s taxonomy, the higher learning of students. So, in itself, clickers... is only a tool. But it needs to be complemented with good conceptual questions in order to make it work.
(Teacher-candidate E)

**Educational Technology as a Tool for Promoting Teacher-Candidates’ Technological Capacity**

The last goal of the study was to examine if modeling the deliberate use of educational technologies in the Physics Methods course can increase teacher-candidates’ capacity for pedagogically effective technology use. This was important, as educational technologies change at such a rapid pace that technologies of today might become obsolete tomorrow. However, the pedagogical practices behind effective technology use will still remain relevant. This emphasizes the importance of helping teacher-candidates develop the capacity for learning new educational technologies and implementing them successfully in their classrooms. One indicator for this process is teacher-candidates’ willingness to view themselves both as learners and as teachers:

I’m there as a teacher, (pause) but I’m also there as a student. Conversely, they’re there as a student, but they’re also there as a teacher. That doesn’t mean they’re teaching necessarily, teaching me. They’re teaching each other... You’re always a student-teacher, regardless of whether or not, what your position says. The-the moment you step out, and you meet someone, you now are both a teacher and a learner. (Teacher-Candidate F)

The observations of the pedagogical practices enacted by the teacher-candidates during the practicum also show that most of them have utilized the technologies discussed during the Methods Courses in their teaching. Interestingly, since not all of these technologies were available to them during the practicum, teacher-candidates were able to figure out alternatives pedagogies. For example, instead of using clickers, a number of them used coloured cards to
promote student voting on conceptual questions, while others used virtual classroom response systems that utilized students' digital devices – cell phones, iPads, etc.

**Conclusions and Future Directions**

In order to prepare students who are ready to face the challenges of the 21st century, we have to educate teachers who are capable of technology-empowered deliberate pedagogical thinking. While common wisdom says that the only person who likes change is a baby with a wet diaper, we have to prepare teacher-candidates to be ready for the technological changes of the 21st century. And if we want to ensure that these changes produce desired pedagogical impact in our STEM classrooms, STEM teacher-candidates should develop the capacity for technology-empowered deliberate pedagogical thinking.

This development should begin in teacher education programs where teacher-candidates acquire the necessary TPCK as well as the attitudes and capacities that will influence their future teaching. In order to achieve that, we propose modeling technology-empowered deliberate pedagogical practices in STEM methods courses. Teacher-candidates should have an opportunity to experiences these practices as learners and as future teachers and realize the value of subject-specific educational research for their own practice. Teacher-candidates should also be encouraged to reflect on their own engagement with technologies and consequent learning. Moreover, as technology-enhanced teaching is much more flexible than traditional lecture-driven instruction, it requires teacher-candidates to possess deeper TPCK. Teacher education programs should focus on its development through allocating sufficient time to STEM methods courses and teacher-candidates' deliberate and research-informed engagement with subject-specific technologies. This engagement should take place in subject-specific courses and should be guided by the available educational research.

This paper suggests that in order to address the research-practice gap in STEM teacher education we have to actively engage teacher-candidates in the practices we want them to enact in their own classrooms. Our research has shown that this engagement increases teacher-candidates' TPCK, improves their attitudes about the role of educational technologies in STEM education, and empowers them to open up to using these technologies to promote student learning in their own classrooms.

While our study focused on the year-long teacher education program, it would be interesting to investigate how teacher-candidates' experiences with technology during the program impact their teaching during the first years after graduation. It is also important to examine how the community of technology-empowered deliberate educational practitioners formed in the methods courses...
can be extended beyond the teacher education program in order to reduce or not close the research-practice gap in STEM education.

References


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PRESENTATIONS
DEVELOPING A DYNAMIC INSTRUMENT FOR ASSESSING TEACHERS' PROFESSIONALLY SITUATED KNOWLEDGE IN GEOMETRY

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Radford University      University of Windsor
Radford, VA             Windsor, ON

Abstract
In this paper we present an interactive dynamic instrument designed to assess mathematics teachers' professionally situated knowledge of geometry. The instrument was focused on the concept of the area of a trapezoid and included GeoGebra files representing samples of students' thinking with a set of follow up questions for teachers to respond. These questions were designed to measure teachers': 1) geometric subject matter knowledge related to the area of a trapezoid; 2) knowledge of students' common challenges and conceptions; 3) ability to ask diagnostic questions; 4) knowledge of applicable instructional strategies and tools; 5) ability to extend understanding of a geometric problem. The original instrument was designed using Delphi methodology. The rubrics for evaluation of teachers' responses were developed using grounded theory approach to identify characteristics and to define different levels of geometry teachers' development of their professionally situated knowledge. The purpose of this instrument is to understand teachers' background in order to design professional development (PD) opportunities most appropriate for a given population of teachers.

In this paper we describe the Trapezoid Instrument and discuss corresponding rubrics designed to evaluate responses measuring geometry teachers' subject matter knowledge and pedagogical content knowledge related to the area of a trapezoid (Shulman, 1986). We adopted the instrument to address the concept of the area of a trapezoid from a Manizade and Mason (2011) study that was conducted using Delphi methodology. We included GeoGebra files representing nine samples of students' thinking with the set of follow up questions for teachers to respond (see Table 1, and Table 2). These questions were designed to measure teachers': 1) geometric subject matter knowledge related to the area of a trapezoid; 2) knowledge of students' common challenges and conceptions; 3) ability to ask diagnostic questions; 4) knowledge of applicable instructional strategies and tools; 5) ability to extend understanding of a geometric problem. The samples of students' thinking were collected over years by the researchers and were divided into three types of approaches (Manizade & Mason, 2014) for developing area of a trapezoid formula (i.e., Type 1 decomposing, Type 2 using transformational geometry, and Type 3 enclosing the trapezoid and subtracting areas). Once teachers were introduced to the samples, they were prompted to interpret the student work and then provide diagnostics of student conceptions. Next teachers
were asked to propose appropriate instructional actions for each of the corresponding samples of student thinking presented in the instrument. Teachers also rated the appropriateness, clarity, sophistication, and limitations of student strategies (Manizade & Martinovic, 2014).

For each item we developed dynamic interactive GeoGebra application (see Table 1, and Table 2). By exploring GeoGebra files teachers had opportunities to interact with the outlined student approaches for deriving the formula for the area of a trapezoid prior to making their pedagogical and mathematical conclusions related to students' thinking. In order to illustrate instrument’s structure and composition we are presenting two samples from the initial Trapezoid Instrument (see Table 1 and Table 2). The final version of the Trapezoid Instrument consists of selected six different samples of students’ thinking about deriving formula for the area of a trapezoid (Manizade & Mason, 2014). Three out of six hypothetical approaches/samples presented in the instrument were generalizable for all trapezoids, whereas the other three are were limited to special cases of trapezoids, such as isosceles and right trapezoids.

Sample Items: Trapezoid Instrument

In the first example the student finds midpoints of two non-parallel sides of a trapezoid and constructs a perpendicular line through the midpoints. This construction creates two right triangles that are then rotated 180 degrees around the midpoints. Unlike student strategy presented in ITEM Y this method is not generalizable to all cases of trapezoids.

Table 1. Samples of students’ thinking using Type 2 strategy, Items X of the Trapezoid Instrument

<table>
<thead>
<tr>
<th>ITEM X</th>
</tr>
</thead>
<tbody>
<tr>
<td>When presented with the task of developing a formula for the area of any trapezoid using formulas for areas of a rectangle and/or a parallelogram and/or a triangle in his high school geometry class Tom developed the diagrams as a strategy for deriving the formula for the area of a trapezoid described by the following GeoGebra file <a href="http://tube.geogebra.org/student/m420201">http://tube.geogebra.org/student/m420201</a>.</td>
</tr>
</tbody>
</table>
a. Based on the Geogebra file attached, describe Tom’s thinking. If he were to complete the formal derivation of the area formula in his diagrams, would this method work for any trapezoid? Why, or why not?

b. If this student’s approach presents a misunderstanding, what underlying geometric conception(s) or understanding(s) might lead Tom to the error presented in this item?

c. If this Tom’s approach presents misconception or misunderstanding, how might he have developed the misconception(s)?

d. What further question(s) might you ask the student to understand his thinking?

e. What instructional strategies and/or tasks would you use during the next instructional period to address Tom’s misconception(s) (if any presented)? Why?

f. If applicable, how would you use manipulatives or technology to address this student’s misconception or misunderstanding?

g. How would you extend this problem to help Tom further develop his understanding of the area of a trapezoid?

In the next sample item the student created a midpoint J on one of the non-parallel sides of a trapezoid ABCD and constructed triangle ABJ. Then he rotated this triangle around point J and created a new triangle IBC. Then he calculated the area of triangle IBC (see the sketch below).

Table 2. GeoGebra file included in the Item Y: Student’s Type 2 transformational strategy

ITEM Y

… Jason developed the diagrams as a strategy for deriving the formula for the area of a trapezoid described by the following GeoGebra file. http://www.geogebratube.org/student/m420191
Development of the Rubrics

To evaluate each item of Trapezoid Instrument we developed rubrics using grounded theory methodology. The rubrics were designed to discriminate between four different levels of teachers' professionally situated knowledge and its sub-components. These rubrics were modified three to four times each and were refined to differentiate between levels of teacher competencies through a reflexive process of linking rubrics to collected sets of raw data from 39 teachers (see Table 3).

The developed rubrics allowed us to identify different levels (0 to 4, 4 being highest of an expert Geometry teacher) of teacher responses for each of the aforementioned dimensions. Table 3 contains an example of a rubric and sample teacher responses for the corresponding levels for the second dimension of the teachers' professionally situated knowledge: Knowledge of student challenges and conceptions.

Table 3. Rubric used to evaluate teacher’s knowledge of student challenges and conceptions

<table>
<thead>
<tr>
<th>Level</th>
<th>Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Teacher is able to identify:</td>
</tr>
<tr>
<td></td>
<td>A. A student’s limited conception of a trapezoid (e.g., isosceles, right)</td>
</tr>
<tr>
<td></td>
<td>B. A student’s limited strategy/method (e.g., using only decomposition; composition is basic; strategy that may not always work—decomposing trapezoid into a rectangle and two triangles, transformation may not always work, while enclosing and subtracting excess will always work) OR</td>
</tr>
<tr>
<td></td>
<td>C. A special case potentially resulting in a limited or wrong formula.</td>
</tr>
<tr>
<td></td>
<td>D. A student’s developmental level in geometry using Van Hiele theory of trapezoid concept OR</td>
</tr>
<tr>
<td></td>
<td>E. A student’s developmental level in geometry using Van Hiele theory with respect to area concept (0-not understanding area; 1-basic understanding of adding units; 2-if the shapes match then their areas are equal; 3-if you re-arrange them they will still be the same; 4-using transformational geometry or simple Euclidian proof to claim equal areas).</td>
</tr>
<tr>
<td></td>
<td>F. A student potentially developing these challenges due to the limited experiences with different types of trapezoids or tools used or lack of motivation.</td>
</tr>
</tbody>
</table>
| 3     | Teacher is able to identify:
A. A student’s conception of a trapezoid as being limited (e.g., to isosceles trapezoid, to right trapezoid).

B. A student’s limited strategy (e.g., using only decomposition; composition is basic; strategy that may not always work—decomposing trapezoid into a rectangle and two triangles, transformation may not always work, while enclosing and subtracting excess will always work) OR

C. Special case potentially resulting in a limited or wrong formula.

D. Student potentially developing these challenges due to the limited experiences with different types of trapezoids or tools used or lack of motivation.

Teacher is able to identify:

A. A student’s conception of a trapezoid as being limited. However teacher does not specify how is it limited, nor proposes any counter-examples in their explanation.

F. A student potentially developing these challenges due to the limited experiences with different types of trapezoids or tools used or lack of motivation.

Teacher recognizes that there is a misconception (if any) in student thinking but does not provide sufficient explanation of the actual misconception OR his/her explanation is mathematically incorrect.

I. The mathematical terminology is incorrect/poor OR

H. The main focus is on the formula, algebra and counting the area units.

J. Did not understand the question OR

K. Did not provide an answer OR

L. Thinks that correct approach is wrong (when it is correct) and correct (when it is not) OR

M. The explanation presents a mathematical error OR

N. Does not address geometrical aspect but focuses only on algebra.

Trustworthiness and Rigor

The initial instrument was developed using Delphi methodology, which established the instrument’s content validity based on the experts’ judgments that the content of the items was consistent with what was being measured. The researchers asked the panel of experts in geometry education to analyze each item with respect to the categories of the original table of specifications. They re-examined the data to identify evidence from the expert responses confirming the validity of each proposed item. The experts’ quotes were then combined in order to better substantiate the items. The investigators also reevaluated all data to search for disconfirming evidence in terms of the validity of each item. This procedure increased the content validity of the instrument as a whole. All of the experts on the panel had previous experience in designing survey instruments as well. Finally, the issues of content and construct validity were addressed by the extensive literature review.

The researchers addressed trustworthiness in terms of credibility, transformability, dependability, and confirmability. The trustworthiness was enhanced through: (a) member checking, as part of the Delphi requirements; (b) peer debriefing; (c) triangulation over time, through all three rounds of
data collection and analysis; (d) detailed description and reporting of the research process; and (e) research notes, which recorded the development of each item and the emergent categories of data.

In addition to further confirm the validity of the instrument in our current study, we tested and established high correlation between this new Trapezoid Instrument and the well-known, valid, and reliable assessment tools in the field of mathematics education, such as IQA observation rubrics, as well as Usiskin’s Van-Hiele test (Matsumura, Slater, Wolf, Crosson, Levison, Peterson, Resnick, & Junker, 2006; Usiskin & Senk, 1990). Both validity and reliability of the instrument were confirmed.

Discussion and Conclusions

The instrument and rubrics described in this paper were designed to measure teachers professionally situated knowledge in geometry and its specific aforementioned components. The purpose of the instrument was to gain information about teachers’ background in order to design professional development opportunities most appropriate for a given population of the teachers. Our instrument was developed with a focus on a geometric content commonly taught at the middle grades as well as high school geometry curriculum. The dynamic software, GeoGebra, files were developed and imbed into the instrument as samples of student thinking for teachers to analyse. The final version of the instrument included selected six samples (some mathematically accurate and some presenting students’ difficulties) of student thinking following with questions for teachers. The validity and reliability of the instrument were established in the study.

We assume that it is important to gather information on teachers’ background prior to providing professional development to them. We propose to do so, by strategically selecting a small sample of targeted topics (e.g. area of a trapezoid) that all secondary teachers have to know. Then we recommend designing short instruments similar to the one described in this paper. We argue that these types of instruments have capacity to produce information on mathematics teachers’ background and can help teacher educators to create profiles of the teachers’ knowledge. This may allow designing PD targeting needs of a particular group of teachers. We do not intend for these instruments to be used to evaluate teacher backgrounds for purposes other than design of the appropriate and effective PDs.

References


This paper reflects on a unique activity in a geometry course delivered to elementary mathematics education students studying in their first year of the undergraduate study. There are three reasons why this activity is granted as valuable and unique: (1) the motivation for the activity, (2) the structure of the activity, and (3) the ways students benefitted from the activity. Each of these reasons will be addressed in the following sections.

Learning geometry is accepted by many students as a rote memorization of geometric facts and a procedure of rapid application of these facts into certain contexts. However, literature suggests that geometry is strongly based on exploration, or investigation, and developing a rigorous understanding of concepts through formal use of geometric language (Johnston-Wilder and Mason, 2011; Kidron and Dreyfus, 2014; Leikin and Grossman, 2013; Sinclair, Pimm, Skelin, and Zbiek, 2012). While talking about the motivation for the activity, a brief connection of geometry and formal use of language will also be provided below.

**Motivation for the activity**

Curricular guides from various countries put emphasis on conceptual understanding of geometry and formal language use in mathematics—here, we take mathematics as a broader term covering geometry (Ontario Ministry of Education, 2005; Sinclair, Pimm, Skelin, and Zbiek, 2012; TTK, 2013). Sinclair, Pimm, Skelin, and Zbiek (2012) claim that, “[l]earners often struggle with ideas about geometry” (p. 1), and it is widely pronounced that the geometry is a strand in school mathematics, which is “challenging for teachers to teach” (p. 1). They support their claims by posing some questions addressing content and formal language of geometry. We will delve into the importance of exploration and formal language use in geometry for the sake of this paper scope.

In the Ontario Grade 1-8 Mathematics Curriculum Guide (2005), it is suggested that “students learn to recognize basic shapes and figures, to distinguish between the attributes of an object that are geometric properties and that are not, …” (p. 9). It is important for us to draw readers’ attention on two concepts in this quote: recognizing and geometric properties. How are students supposed to recognize the figures and shapes? What properties are called geometric and how are these properties connected to the recognition of figures? The verb to see English and gormek in Turkish is also used to describe the recognition of figures and identify their properties. Johnston-Wilder and
Mason (2011) connect seeing to visual perception and support their connection by demonstrating a couple of geometric illusions. But, what makes seeing is more possible and easier? Although the topic is essential to learn and teach geometry, it is too broad to discuss in a conference proceedings. Therefore, we take our attention back to the exploration and the use of formal language in geometry.

Sinclair, Pimm, Skelin, and Zbiek (2012) argue that one of the big ideas related to geometry is that, “[c]lassifying, naming, defining, posing, conjecturing, and justifying are codependent activities in geometric investigation” (p. 55) and put the relationship between naming and properties by stating, “draws attention to properties and objects of geometric interest” (p. 56). Similarly, Turkish curriculum guide published by TTK (2013) suggests that the use formal language of geometry, alongside with visual perception, is one of essentials in learning geometry.

However, elementary and high school students as well as undergraduates often lack the skill of using formal language. It was realized that students, taken the mandatory geometry course in the Bayburt Faculty of Education, misused some concepts: For example, median and height of a triangle were being used interchangeably. The ill-use of basic geometric concepts often leads many students to misunderstand the concepts or misinterpret the problems even if they develop a proper understanding of concepts. The emergence of this problem encouraged the course instructor, also the second author of this paper, to develop an activity. The following section will describe the activity and its structure and provide some tasks developed for the activity.

**Structure of the activity**

Once students’ misuse of formal geometry language in the course and in their papers was identified, the course instructor, who is also the second author of this paper, designed an activity to engage students in exploring geometric concepts with GeoGebra. The first author of the paper was assisting the instructor and observing students in the course for her master study.

Forty six tasks were developed, because 46 was the number of the students taken the course, and posted on a wiki space, which was effectively being used during the course. Then, students were asked to select one task to work on and declared their decisions to their friends to avoid the selection of the same task by another. The selection process was on first-come-first-served basis. Since wiki space records the exact timing for each selection, it was found fair to use it for that purpose.

The reason not to assign one task for each student but ask them to select their own task was to encourage them to review the tasks and evaluate them before posting their decisions. It was
expected by the instructor that each student would review each task and possibly give some try or at least reflect on how the task could be performed.

In all tasks, they were supposed to create a different GeoGebra dynamic worksheet, demanding various levels of exploration and formal use of geometric language. Some of the tasks were asking to create the same worksheet but written in different wording. Some tasks are as follows:

- Draw a KLMN parallelogram. Find intersection point of angle bisector of KLM interior angle and median of LM.
- KLM draw a triangle whose vertices are on the circle. Find the intersection point of angle bisector of LMK external angle and perpendicular bisector of KM.
- Draw a KLMN parallelogram. Find intersection point of height of KL and perpendicular bisector of LM.
- Draw a pentagon whose vertices on the circle. Find the intersection point of perpendicular bisectors of consecutive two sides of the pentagon.
- Draw a regular ABCDE pentagon. Then draw diagonals AD, AC and perpendicular bisector of these diagonals. If you call intersection point of perpendicular bisectors as F, find type of ABFE quadrilateral.
- Draw a quadrilateral whose vertices are not on a circle and diagonals are perpendicular to each other.
- Draw a parallelogram with diagonals intersecting at an angle of 30 degrees.

Once they completed their tasks, they were supposed to upload their GG files to the same wiki space to be graded. Uploading this space created an opportunity for them to discuss their work with peers, before grading was started. They were continuously encouraged to communicate with each other through wiki space so that they could get a chance (1) to receive feedback from their peers and (2) to engage in talking about the activity after the classroom. The latter was particularly important for us because through that interaction, we could be aware of their weakness in using formal geometry language.

Moreover, we could be able track their improvements in this regard because many realized about their misuses through these communications and could get a chance to fix their mistakes. The following section will conclude the paper by reflecting on the activity and by presenting our findings about students’ ways of benefiting from this activity.

**Conclusion**

This paper reports on a class activity developed to remedial students’ lack of use of formal geometric language and improve their conceptual understanding. In order to understand whether
students improved their understanding and use of formal geometric language, their work was analyzed and some informal communications were performed.

The first thing mentioned by the students was the challenge to select the easiest task. They declared that they worked very hard to review as many task possible because they knew that one of their peers would have selected one if they were late. The selection, of course, demanded to understand –at least to a certain degree -the task and to develop an insight on how the task could be done.

The second point they talked about was to realize how much misuse of language they had developed during their previous schooling. Their talk revealed that they could get more successful in understanding geometric relationships once they realized the difference among certain concepts, such as median and height.

The third theme emerged from our conversation was the importance of exploration. They stated that rather than applying some procedures to certain test questions, exploration of a task helped them to uncover the ideas behind procedures. Furthermore, almost all of them pointed out that they found geometry as joyful topic as opposed to what they had believed before.

Our personal experience after completing the activity is that students enjoy and understand better through activity based learning and communications among peers, which is not a surprise, as stated in literature. Although some of them demonstrated some resistance against this approach, it might be because of their previous habits of learning. Following this activity, another in-classroom activity was designed and students were engaged to explore various concepts in groups.

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AFFORDANCES OF INFORMATION COMMUNICATION TECHNOLOGY USAGE FOR COLLEGE FOUNDATIONAL MATHEMATICS STUDENTS

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Abstract
Although accepted into first year diploma programs, not all students are equally prepared or ready for learning in Ontario Colleges. Students placed into foundational mathematics courses may find several factors challenging, including one or more of the following: lack of interest in the material, poor motivation, learning difficulties, or workload. Use of information and communication technology (ICT) can prove supportive by affording a two-way communication and immediate feedback between teachers and students, the ability to present work anonymously, the allowance for students to teach, and the flexibility of giving students the option of determining their own learning pathway. Through multiple theoretical lenses, this paper will examine the learning gains afforded by ICT and in particular, the use of pen-based computing with tablet PCs and DyKnow interactive software.

Introduction
“Proper use of technology can change adult developmental mathematics education for the better. Considering the efforts put forth by adult learners, the special needs they have which have affected their opportunities to learn, and the substantial population whose futures may be at stake, mathematics educators cannot afford to ignore this work.”

(Garrett, 2010, p. 211)

At many Colleges in Ontario, students may be required to take a standardized test and based on their performance, may be placed into foundational (developmental/remedial) courses. Students placed into foundational mathematics courses are described as being under-prepared due to previous study experiences, may have poor habits or study skills, diagnosed as being on the learning disorders spectrum, or simply be out of practice in a learning environment (Blockin, 2008; Morris, 2011). In my experience, adult learners who are recent immigrants, have received instruction in another language, or have simply been away from an academic setting for a prolonged time, account for a significant proportion of this grouping. As foundational mathematics courses need to engage unique learners with diverse backgrounds, technology offers the ability to provide for differentiated instruction (Li & Edmonds, 2005). The function of technology use has evolved from providing assistive devices to at-risk learners to supplementing instruction and enhancing performance for students of all abilities (Edyburn, 2003).
For my study, technology plays a vital role as it provides a learning workspace that cannot be accomplished in a traditional teacher-centered classroom. ICT allows access to resources, provides the tools to facilitate, and the ability to immerse the individual in authentic learning. As students explore mathematics with technology, they can evolve, construct, and assimilate their own knowledge (Hannafin & Land, 1997). Teachers can organize related themes to allow multiple entries and connections, but students select the pathway to achieve their understanding. Interacting in group activities, they share their knowledge, or build on information provided either by the teacher or fellow-students (Carruthers, 2011). With the improvement of ICT, the potential “to expand, strengthen, and create efficiencies in the delivery of developmental math practice” (Epper & Baker, 2009, p. 3) has advanced. Though technology may increase student engagement due to the consistent novelty of ever-changing devices, instructional design needs to be based on established learning theory (Ally, 2004).

**Applicable Learning Theories**

According to Martinez (2010), behaviourism is the study of how humans learn by examination of “behavior rather than thoughts or feelings” (p. 6). Learning, to a behaviourist is conditioning: the subject (human or animal) responds to a stimulus or event with changes in observable behaviour. Psychologists classify conditioning, using positive or negative reinforcement to modify performance, from two historical paradigms: “classical”, from the work of Ivan Pavlov (p. 7), and “operant”, from that of B.F. Skinner (p. 14). In traditional forms of instruction, information is controlled by the teacher and transmitted to passively receiving students and due to repetition (e.g., rote learning, testing) learning becomes habitual (Case & Bereiter, 1984). In operant conditioning, according to Skinner (1974), “the behaviour itself is said to be strengthened by its consequences, and for that reason the consequences themselves are called ‘reinforcers’” (p. 44). A behaviour is more likely to occur repeatedly if the consequence is reinforcing, such as rewards or teacher approval given for correctness (Ertmer & Newby, 2013; Martinez, 2010). In teaching, an observable change in behaviour “indicates whether or not the learner has learned something” (Ally, 2004, para. 11).

In mathematics teaching, behaviourist practice relates to procedural learning where basic skills must be mastered before more advanced activities can be built upon them (Klinger, 2011). Exposition methods of teacher demonstration followed by student practice or rote memorization of mathematical facts can be considered classical conditioning. Most of us will remember learning mathematics this way, where teachers demonstrated the method and we as students practiced repeatedly. Unfortunately, in many cases, the knowledge obtained was quickly lost if not continuously used. Clearly, practice develops “functional fluency” (Klinger, 2011, p. 11) however, learning without “structured, regularized or intrinsic opportunities to promote understanding” (p. 11)
negates the operant conditioning benefits of building on prior and life experiences. However, one cannot negate all aspects of behaviourism in teaching college foundational mathematics, as the requirement for a correct answer demonstrates a “functional fluency that can only be found as a result of practiced ease” (p. 11).

Noam Chomsky’s concept of “nativism” specifies the “brain has some innate knowledge” (Martinez, 2010, p. 28) with which humans are born. In light of advancing knowledge of brain function, a shift from learning solely based on behaviour modification to one recognizing the influence of mental processing brought cognitivism to the forefront (Klinger, 2011). According to Ausubel (1969) the “distinctive features of a cognitive learning theory” deals most importantly with “meaningful as opposed to rote learning” (p. 331). In addition, since “computers can perform cognitive operations”, then “regulative thoughts could no longer be denied to humans” (Bandura, 2001, p. 2). Cognitivism stresses the “intentional action deriving from a learner’s mental states” (Klinger, 2011, p. 12). Jean Piaget’s theories of cognitive development recognized that knowledge was assimilated or accommodated by a “psychological tension between beliefs and experience” (Martinez, 2010, p. 197). If the knowledge we currently possess is similar to our experience, there is no requirement to change our beliefs, therefore we “assimilate” this new information. Accommodation is required when new knowledge does not match our current conviction or it is unknown and cannot be explained. Simply put, we need to “change our mind” (Martinez, 2010, p. 198). Like homeostasis in a biological feedback loop, the system is out of balance, and this disequilibrium requires restructuring to make sense of conflict. Learning, for a cognitive psychologist, “involves the use of memory, motivation, and thinking” (Ally, 2004, para. 12) and “emphasizes the salient role of the learner’s existing structure of knowledge” (Ausubel, 1969, p. 331). Albert Bandura (2001) recognizes two pathways directing psychological discipline, one theorizing on the “microanalysis of the inner workings of the mind in processing” and the second focusing on the “socially situated factors of human development, adaptation, and change” (p. 5). Relating to the explanation given above, those cognitive processes studied for the first path relate to how information is coded, processed and retrieved, in manners that are largely “disembodied from interpersonal life” (p. 5). Bandura’s social cognitive theory analyzes human function as “interdependent, richly contextualized, and conditionally orchestrated” (p. 5) within complex societal subsystems.

In mathematics, research regarding effective methodologies for solving problems through the reduction of cognitive load, relates to cognitivism theory (Kalyuga, Chandler, Tuovinen, Sweller, 2001). On the other hand, techniques such as scaffolding and modelling of materials to develop active processes to benefit learning mathematics for all in the classroom, relates to social cognitivism (Henningsen & Stein, 1997). Doing mathematics is often associated by examining
classroom engagement with “cognitively demanding mathematics tasks” which require “high-level reasoning and problem solving” (p. 526).

Constructivism centralizes the learner, building their own reality based on personal experiences gained from “observation, processing, and interpretation” of the world surrounding them (Alley, 2004, para. 13). Educators using constructive approaches to design instruction activities, some of which include: (a) learning must be relevant to real-world tasks, (b) the learner controls and mediates their learning, and (c) the instructor provides authentic tasks and guides the student by helping them to focus on their learning strategy (Jonassen, 1991). In terms of making meaning, constructivists align themselves with the theories of Piaget and Vygotsky in that the process is “emphasized both as an individual mental activity and a socially interactive exchange” (Lee & Lin, 2009, p. 59). With greater focus on social learning, the theories of Lev Vygotsky have gained prominence (Martinez, 2010), and form the foundation of social constructivism. Vygotsky (1978) defined the Zone of Proximal Development (ZPD) as the gap between individual problem-solving skills and the level possible through collaboration with an adult facilitator or a more capable peer. This social interaction is integral to the development of new pathways for thought and strategic mapping.

For mathematics, Klinger (2011) reinforces these concepts as learners are “discovering relationships between new information and their knowledge representations and constructions of reality” (p. 12). Constructivism gives the learner an active role in information gathering, the cognitive internal process of knowledge storage, and thereby builds understanding. In social constructivism, knowledge is shared with peers, extended, and “should be viewed as both a process of active individual construction and a process of enculturation into the mathematical practices of wider society” (Cobb, 1994, p. 13). However, Klinger (2011) expresses concern that mathematical form, solution strategies, and answer require specific conventions (rules) be followed as “students must know what to write and how to write it” (p. 13). In addition, he suggests that constructivism will require self-directed learners to have basic skills. This is problematic, as not all mathematics learners are self-directed and some may be lacking in basic skills that will impact their future learning. Successful social constructivist environments require partakers to have some ability to maintain personal self-regulation, reflection and abstraction (Murphy, 1997). With learners at various different stages of understanding, it can be challenging for the instructor to provide an environment to suit individual needs. The challenge of diverse classroom situations can result in reversion to teaching in behaviourist or cognitivist configurations (Klinger, 2011).

A final theory for consideration is one developed by George Siemens and Stephen Downes (Duke, Harper & Johnston, 2013). Although debate exists as to whether it is a “learning theory or
instructional theory or merely a pedagogical view” (p. 4), it is relevant to the discussion here as it pertains to learning in the digital age. Building from a “three-fold view of epistemology” (behaviourism, cognitivism and constructivism), Siemens (2008) credits Stephen Downes with “a fourth [theory]: the view of knowledge as composed of connections and networked entities” (p. 10). Siemens (2004) maintains that previous learning theory relates to how learning occurs, but does not assess the “value of what is being learned” (para. 12). As available knowledge is now so abundant, the meta-skill of evaluating information to determine its importance is critical. Recognition of patterns and connections is required so that only essential information is synthesized is a desired skill set for today’s learners. “Connectivism is the integration of principles explored by chaos, network, and complexity and self-organization theories” (Siemens, 2004, para. 24). Chaos determines that meaning exists, as all things are connected by networks. The challenge for the learner is to work through the complexity of the information flow to find useful patterns which may be hidden. Pattern adjustment may be required if the original parameters used for decision making are changed. Connectivism is the ability to self-organize information into useful patterns and form connections between information sources.

In education, connectivism is “learning situated across space and time” (Kumpulainen & Sefton-Green, 2014, p. 7) nestled in social spaces, classrooms and communities. Siemens (2008) views knowledge as obtainable from networks, and fundamental to this theory is how a connection “exists between two entities when a change of state in one entity can cause or result in a change in state in the second entity (Downes, 2014, para. 13). Increased mobility means that understanding can be generated not just in a formal classroom but also in distributed settings (Waters & Gasson, 2007). As long as the learner is able to integrate and connect with their information sources, knowledge can be maintained external to the knower. As the learner needs new information, they reconnect to their network for update and modification (Siemens, 2008). “For the learner to be connected to this outside knowledge is more important than his or her existing state of knowing.” (Duke et al., 2010, p. 6). A personal network system has advantage in that critical decision making is based on up-to-date, relevant knowledge filtered from the diversity of opinions forming the network. Siemens (2006) finds several important “change drivers” impacting modern learning theory. Of those listed, the most significant to this study are the emerging philosophies of knowing, expectation of students and the development of technology. He finds a “growing disconnect” (Siemens, 2008, p. 7) between experiences, activities and tools found in the classroom with those used regularly outside of the classroom. Some authors are speculative about educational roles in connectivist environments. Duke et al. (2010) suggest there is no value in having access to information if it is beyond the level of the learner. Siemens (2008) suggests that learning requires guided exploration, rather than unguided, to increase learning gains. He finds the metaphor of a
“curator” (p. 17) to resonate with this thinking. The curator (instructor) provides a workspace and guides learners to explore possibilities by selecting their pathway through the network.

Klinger’s (2011) focus on mathematics finds previous learning theories lack breadth in today’s digital age, and thus he employs connectivism. When working with students demonstrating math-averse behaviours in post-secondary settings, he finds that by “exploiting the properties of network connectivity” (p. 15) they learn more effectively. His model suggests that strong connections should first be developed between language and mathematical processes (e.g. explanation of steps) from existing prior knowledge. In this way, learning is shifted from mathematical procedure to conceptual understanding, as explanation fluency increases. Novel information results in new connections, some of which may not agree with past knowledge. At a critical point, this inequality will require self-reorganization with “cognitive phase transitions” that “spontaneously yield flashes of emergent deeper understanding” (p. 16). With increasing understanding, a sense of empowerment will develop, to the point where the learner will be able to self-direct their own learning. Klinger’s theory demonstrates an attempt to explain ‘ah-ha moments’ in mathematics. The theory of connectivism has elements of all the previous theories to illustrate the present, then builds on them to advance the future of learning theory.

**Instructional Design Related to Learning Theory**

Smith and Ragan (1999) define instructional design to mean “the systematic and reflective process of translating principles of learning and instruction into plans for instructional materials, activities, information resources, and evaluation” (p. 2). Using the discussed learning theories, this paper concludes by reviewing how the design of online resources incorporates either single or multiple attributes of these theories. The use of the description ‘online learning’ here refers to multiple terminologies (e.g., Web-based learning, Internet learning, computer-based learning), implying the use of ICT to provide an instructional medium between students and teachers (Ally, 2004). From my teaching practice over the past 7 years, students in college foundational mathematics courses use pen-based computing on tablet PC’s to take hand-written notes using DyKnow Vision software (Carruthers, 2011). The instructor provides a note framework and opens a DyKnow session. Students are invited to log in, thus enabling a virtual workspace. At the end of the session, students save their notes which can be accessed via the internet. The software allows a medium for collaboration and interaction between students and teacher. The details of some of its features will be related to previously described learning theory.

Using a behaviourist online learning framework, designers consider the most critical instructional elements relate to how stimuli are arranged, and reinforcement organized, when providing resources (Ertmer & Newby, 2013). Pre-testing is used to determine where instruction should begin.
and expectations of outcome are clearly outlined. Online activities are provided in a sequential manner in such a way as to master early steps (e.g., fact recall) before more moving onto more complex understanding (Ally, 2004). Reinforcement in an online environment often takes the form of testing, however intermittent informative feedback can also provide the desired learning outcomes (Ertmer & Newby, 2013). In our classroom, a note framework is provided in sequential order; however, students have the ability to advance according to their preference. The software allows multiple avenues for feedback in teacher-student, student-student, and student-teacher directions. Students can anonymously: (a) raise a ‘red-flag’ to teachers when understanding is lost, (b) submit individual panels of work electronically, or (c) immediately respond to teacher-generated polling questions. For foundational mathematics students, a synchronous learning environment gives multiple ways of providing stimuli and responsive feedback.

From a cognitive online learning framework, emphasis is placed on “how to design instruction so that it can be readily assimilated” (Ertmer & Newby, 2013, p. 53). Consideration of presentation ranges from how information should be placed on-screen to maintain attention (Ally, 2004) to research on the impact of multimodal learning to reduce cognitive load (Mayer and Moreno, 2003). Unlike the stimulus/response structure built into behaviourist design, resources developed from a cognitivist viewpoint attempt to encourage the learner to use “appropriate learning strategies” (Ertmer & Newby, 2013, p. 52) by focusing on organization and goal setting. In addition, introductory activities are designed to make connections to prior learning, resources are limited to manageable chunks, and online mind-mapping tools assist in organization, all supporting the concept of active-learning (Ally, 2004; Ertmer & Newby, 2013). In our classroom, it is the extensive use of the Internet to seek our relevant learning materials, (built into the note framework) which most closely addresses a cognitivist learning environment.

The use of the Internet gives students an infinite learning resource; however, when guided by an instructor, relevance, novelty and interest can be maintained.

Using a constructivist online learning environment, activities that give students the opportunities to build and experience information for themselves is particularly relevant (Ally, 2004). Through a design that is learner-controlled, students construct and reflect upon understanding which is then validated by collaborating and using the strengths of others to negotiate meaning (Ertmer & Newby, 2013). Using tablet PC’s, students write individual solutions to mathematics problems which can then be shared using the DyKnow software through multiple options. The teacher can receive virtual panels sent by students which can be added directly to the note framework, or they can ‘pass control’ to the student giving them the option of solving directly onto the screens of their
peers. When the instructor places students into virtual groups, all can share a single screen to develop a collaborative solution which can be shared with the entire class.

To demonstrate connectivist framework, Armatas, Spratt, and Vincent (2014) describe key features including “the use of e-learning tools to facilitate networking, knowledge sharing, critical consumption of information, and continuous learning” (p. 81). Building on constructivism, knowledge is gained not only by experience but also through personal networks that individuals have developed (or are required to become members of) in course curricula (Transue, 2013). Students not only consume information, but “participate in the knowledge creation process” as part of their learning (Dunaway, 2011, p. 675). Within our mathematics classroom, some aspects of connectivist learning are more difficult to implement due to the nature of the subject matter. Previously, social media (e.g., Facebook, Twitter) were limited by the necessity of typed input, however this barrier can be overcome with picture or video input. Students develop mathematics screencasts (audio and video recording of computer screen) to design learning resources for themselves, their peers, and others who use Web-based instruction. Although these limited examples of networking do not encompass the depth of connectivist theory, our preliminary use of ICT demonstrates a potential for future instructional design.

In summary, the flexibility of online instruction gives the opportunity to experience multiple learning strategies. In the college foundational mathematics classroom, although online resources are designed based on learning theory, it is only in the consistent use of ICT that future possibilities for learning can be envisioned.

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TEACHERS EVALUATING DISSECTIONS AS AN APPROACH TO CALCULATING AREA OF TRAPEZOID: 
EXPLORING THE ROLE OF TECHNOLOGY

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Abstract
This paper stems from a mixed-methods study designed to inspect pedagogical content knowledge of in-service teachers in relation to the area of trapezoid, a common topic in elementary school classes. Based on the provided exemplars, teachers were asked to rate the appropriateness, clarity, sophistication, and limitations of student strategies, and propose possible ways to address students’ misconceptions and difficulties. Their quantitative evaluations and comments of student work were analysed, as well as the technological extensions they thought may help with alleviating some limitations of the existing approaches.

In this paper we describe a mixed-methods study in which we analysed pedagogical content knowledge of in-service teachers. The teachers were enrolled in an online Geometry course, where one of the discussion topics was the Area of a Trapezoid; a common topic in middle grades and high school classes. After discussing different ways to calculate the area of trapezoid and to develop a general formula, the participants were provided with nine fictional exemplars of student solutions (some correct and some with mathematical limitations) and prompted to evaluate and interpret “student” work. These examples were developed using the sample of three types of approaches (Manizade & Mason, 2014) for developing area of a trapezoid formula (i.e., decomposing, using transformational geometry, and enclosing the trapezoid). To collect data, we used the pedagogical content knowledge instrument, Trapezoid PCK Instrument, adapted from Manizade (2006), and Manizade and Mason (2010). Aspects of this instrument gave us insights into the level of geometry content knowledge (i.e., 0-4) that each teacher had. Teachers were also asked to rate the appropriateness, clarity, sophistication, and limitations of student strategies, and propose possible ways to address students’ misconceptions and difficulties. The numeric scores were incorporated into a mean composite score (i.e., (Math Appropriateness + Clarity + Sophistication + Lack of Limitations)/4) that was associated to each student exemplar. The sketches that accompanied exemplars were done in GeoGebra and teachers were encouraged to propose technological extensions of presented approaches.
In this paper we discuss our findings based on three fictional exemplars from the Trapezoid PCK Instrument which all used dissections (Martinovic & Manizade, 2014) in a form of decompositions. These dissections were very simple and sometimes based on a very limited case, such as using an equilateral trapezoid. Using the theorem that any polygon can be transformed into another polygon of the same area by cutting and rearranging the pieces (Frederickson, 2007), the teachers could assume that the proposed dissection may lead to a parallelogram or a triangle; a figure with a known area formula (i.e., using a related and previously solved problem, Polya, 1957). However, finding the area of a concrete trapezoid was not enough and provided examples did not always lead to the general formula for the area of trapezoid.

**Analyses of Exemplars**

The following exemplars of student work were provided to teachers. They were asked to assess student approaches using a four level scale (i.e., Not Appropriate = 0, Somewhat Appropriate = 1, Appropriate = 2, Very Appropriate = 3), assuming that these exemplars were collected in their own class. We provided teachers with a rubric according to which they had to evaluate the Math Appropriateness (how suitable is this approach to generate expected outcome/formula for the area of any trapezoid; Levels 0-3), Clarity (how clear/unambiguous is this student’s strategy/approach, Levels 0-3), Sophistication (how sophisticated/complex is the strategy/approach presented by this student, Levels 0-3), and Lack of Limitations (how limited is this approach, Levels 0-3) of the student approach to developing a formula for the area of any trapezoid.

<table>
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<th>Case 1 Whitney’s approach</th>
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Analysis of Case 1 (dissection). In this scenario the student decomposed a trapezoid into a rectangle and two congruent triangles. Then, she/he added the areas of all three shapes to calculate the area of the trapezoid. This approach could mislead the student in developing a formula; for example, one outcome could be that \( A = 2b_1^2 \). The teacher should notice that this is a special case where not only an isosceles trapezoid is used, but also \( b_2 = 3b_1 \) and \( b_1 = h \). Therefore, having a square as a dissection may mislead the student that the area of trapezoid is double the middle part.
Teachers' evaluations and observations. Almost all teachers stated that this approach is limited and would work only with isosceles trapezoid. One teacher who received a maximum score in knowledge of geometry (i.e., 4), stated: This student’s “approach only works for trapezoids that are akin to isosceles trapezoids. She did not consider all types of trapezoids, but did understand the basic make-up of area and deconstructing a trapezoid.”

There was a significant negative correlation between the teachers’ knowledge of geometry and their evaluation of the approach presented in Case 1 ($r = -.461$, $n = 16$). Teachers with higher level of geometry knowledge as measured by the Trapezoid PCK Instrument gave lower marks to this student’s approach.

### Case 2 Abby’s approach

![Diagram of Case 2](image)

**Analysis of Case 2 (dissection).** In Case 2 student decomposed a trapezoid into two triangles. Then he/she added the areas of these triangles to calculate the area of the trapezoid. This approach would work for any trapezoid, as any quadrilateral could be dissected into two triangles. Noticing that both triangles have the same height is essential for this example.

Teachers’ evaluations and observations. All teachers agreed that this method should work for all trapezoids, but they did not agree on the mark for clarity. The teacher who was at level 2 of geometry knowledge as measured by the Trapezoid PCK Instrument wrote, “Other than the diagram and the brief description provided, there is not much else to critique this strategy. I believe this method is pretty standard and effective however can be difficult for some students to find the height of irregular shaped triangles formed by the diagonals.” There was a weak negative correlation between the teachers’ knowledge of geometry and their evaluation of Case 2 dissection approach ($r = -.114$, $n = 15$), resulting in 1% shared variability between these two variables.
Analysis of Case 3 (dissection). In this scenario a student decomposed a trapezoid into a rectangle and a right triangle. Then she/he added the areas of these shapes to calculate the area of the trapezoid. This is a special case – a right trapezoid for which one base is double the size of the other base. Consequently the area of trapezoid is very easy to calculate as \[ A = \frac{3}{2} (b_1 \times h) \]. Teachers should note that this student may have a problem with a concept of trapezoid and limited skills in calculating the area of simple figures, which will prevent him to find the general area formula.

Teachers' evaluations and observations. Teachers were very critical of this exemplar. They recognized that the student deals with a special case of trapezoid and that not all trapezoids can be dissected into a rectangle and a right triangle. Yet, one teacher whose knowledge of geometry was at Level 1 wrote, “I really enjoyed and think that this approach could be one of the best decomposition we saw in this lesson.” There was a medium negative correlation between the teachers' knowledge of geometry and their evaluation of the approach \( r = -0.464, n = 16 \). Teachers with higher level of geometry knowledge gave lower marks to student in this scenario.

Discussion

Overall Abby’s exemplar had the highest average score on Mathematics Appropriateness (\( M = 2.77 \)) (see Figure 1). The teachers considered all exemplars as clear, as the range of their average evaluations was about one point. The highest average score on Clarity was that of Abby’s (\( M = 2.29 \)), while the lowest was Whitney’s (\( M = 1.85 \)). Abby’s approach was on average evaluated as most sophisticated (\( M = 1.36 \)), while Whitney’s and Paul’s were close to each other, but below .5 score. The average score on Lack of Limitations of the approach provided the largest range, where Paul’s approach scored \( M = 0.53 \) (most limited), and Abby’s \( M = 2.54 \) (least limited).
Figure 1. Teachers' Ranking of Mathematical Appropriateness, Clarity, Sophistication, and Lack Limitations of Students' Exemplars.

Comparison of different exemplars based on the four criteria (see Figure 2) revealed that clarity has little to do with other aspects. On average there was close to .5 points variability in clarity among the three approaches, while variability on other aspects was much greater. Both Whitney’s and Paul’s approach were high on clarity, but low on other three aspects. Compared to other two, Abby’s approach scored highest on all aspects, but its sophistication was not highly valued by the teachers.
Using technology or manipulatives to address students' misconceptions or misunderstandings

Teachers were also asked how they would use technology to address students' misconceptions or misunderstandings. They primarily saw the advantages of dynamic geometry software, such as GeoGebra, for allowing students “to draw every type of trapezoid they think exists,” and to explore all of the different options. In case of Whitney’s approach, “GeoGebra can be used to drag the bases around. After showing that the area on an isosceles trapezoid, drag the base to show that the area stays the same, even though the congruent triangles are not there any longer,” wrote one teacher.

In case of Abby’s approach, the teachers did not recommend using technology as they did not find misconceptions. Only one teacher wrote, “I would still use the GeoGebra software because of the ease of organization in constructing the triangles. This would be very beneficial if the triangles were going to be oddly shaped and hard to measure.”

Probably because the teachers on average considered Paul’s approach as least sophisticated and very limited, many of them recommended use of tangrams and manipulatives, but not technology. One teacher wrote, “Use a drawing program (Geometer’s SketchPad/GeoGebra) and create a page where we have a quadrilateral where two lines are set to be parallel. I would allow students to try and create a shape that is not a trapezoid. Explain why it contradicts with the definition.”
Conclusions

In this paper we especially highlight the value of carefully selected and designed examples which may provide opportunities for teachers to notice details, to make assumptions about students’ thinking, and to offer explanations for possible misconceptions. This allowed us to have some insight into the teachers’ technological pedagogical content knowledge.

It was revealing that when the students used correct/general approaches in the exemplars, the teachers did not find it necessary to recommend use of technology or manipulatives. Technology was mostly recommended because of its dynamic features, its possibility to move and stretch the object while keeping its invariant characteristics intact (moving the vertices of the trapezoid while keeping the bases parallel). Teachers saw this technology affordance as the most important to enable students to go beyond the special cases of trapezoids. However, the term that some teachers were using for GeoGebra and Geometer’s Sketchpad, “a drawing program,” may point to the limited view these teachers have about the affordances of these software programs.

In cases that could point to students’ misconceptions, the teachers recommended use of manipulatives, rather than dynamic geometry programs. Our impression was that the teachers try to make mathematics examples more obvious and try to avoid “confusing” approaches. We believe that it is much to be done to counteract the view of mathematics as difficult, mathematics teaching as consisting of “cookie-cutter” recipes, and students as incapable of doing mathematics in their minds or of working with open problems.

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COLLABORATIVE EXPLORATION OF GEOMETRIC DEPENDENCIES IN DYNAMIC GEOMETRY

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Abstract
The Virtual Math Teams Project (2002-2014) at the Math Forum developed a collaborative-learning environment for mathematics, combining text chat and a multi-user version of GeoGebra. It created curricular activities aligned with Common Core and provides teacher professional development. It has deployed the technology and curriculum with groups of students year after year and analyzed some of the student interactions in micro-detail. Study of how collaborative learning takes place in this GeoGebra-based environment has been used to refine the environment and curriculum. Student teams learn how to collaborate, work online, use GeoGebra, analyze and construct dynamic-geometry figures, think about dependencies among geometric objects and talk about mathematics. This presentation demos the approach and shows how learning about dynamic-geometry dependencies is displayed in a data excerpt.


Keywords. Multi-user GeoGebra, VMT collaboration environment, geometric dependencies, interaction analysis, CSCL, collaborative learning

Research Context and Problem: Guiding Students to Math Cognition
The contemporary fields of science, technology, engineering and mathematics (STEM), in particular, require a mindset that emerged historically within the community of ancient Greek geometers (Heath, 1921). For many people, learning basic geometry still represents a watershed event that determines if an individual will or will not be comfortable with the cultures of mathematical cognition. GeoGebra provides a promising tool for supporting transformational mathematical thinking.

At the Math Forum (www.MathForum.org), we have embedded GeoGebra in an online collaboration environment (Stahl, 2009) and converted it to a multi-user version (Stahl, 2013), so that groups of students can construct and drag figures together, while chatting about what they are doing. To guide student exploration, we have developed a cohesive curriculum focused on the construction of figures with geometric dependencies (Stahl, 2015b)—for use by student teams as well as in teacher professional development. Collaborative GeoGebra is now available for iPad,
The curriculum is in a GeoGebraBook (http://ggbtu.be/b154045), which is not yet multi-user.

In this paper, we demo collaborative GeoGebra and illustrate how we analyze case studies of students engaged over multiple sessions with our online collaboration-learning environment: multi-user GeoGebra, challenging topics and inquiry pedagogy. For the past decade, such analysis of student usage has been driving the iterative design of our approach. We want to indicate how student teams under these conditions display that they are learning fundamental insights about dynamic geometry.

**Theory of Collaborative Dynamic Geometry: Group Cognition and Dependencies**

Learning is often conceived as a change in propositional knowledge possessed by an individual student. Opening up an alternative to this view, Vygotsky argued that students could accomplish knowledge-building or learning tasks in small groups before they could accomplish the same tasks individually—and that much individual learning actually resulted from the earlier group interactions (Vygotsky, 1930/1978), rather than the group being reducible to its members as already formed individual minds. Vygotsky conceived the group interactions as mediated by artifacts, such as representational images and communication media. More recently, educational theorists have argued that student processes of becoming mathematicians or scientists, for instance, are largely a matter of mastering the linguistic practices of the field (Lemke, 1993; Sfard, 2008).

Our pedagogical approach emphasizes collaborative learning through discourse in small groups (Stahl, 2015c). Carefully designed topics guide student exploration and bring in historically developed concepts from the mathematics community. Teachers prepare students before sessions and discuss findings and conjectures in whole-class discussions after the collaborative sessions. The group cognition (Stahl, 2006) that takes place in the group work can lead to learning by individual students in their zones of proximal learning, based on their joint meaning making and task accomplishments.

Our curriculum focuses on learning to construct geometric dependencies (Stahl, 2013) in GeoGebra, a challenging but important skill. While much classroom use of dynamic geometry today merely uses it as a visualization tool, to allow students to drag existing diagrams around, the technology has a greater potential: to empower students to construct their own diagrams, to build their own dependencies into the objects and even to fashion their own custom construction tools. Then they can view Euclid’s propositions as guides to designing and constructing their own interesting mathematical objects, rather than as impersonal eternal truths to be memorized.
Research Method: Sequential Interaction Analysis

The data we collect from hundreds of students using our system each year includes a complete record of their interactions, which we can replay just as it appeared to the students. We also have detailed logs generated automatically.

Figure 2. The interface of the collaboration environment, showing multi-user GeoGebra and text chat.

We use methods of interaction analysis or conversation analysis (Jordan & Henderson, 1995; Schegloff, 2007), adapted to our online math-education setting. This looks at how student groups engage in shared attention, joint representation and intersubjective meaning making. Although we recognize that processes at different levels are inextricably intertwined in reality, we focus methodologically on the group unit of analysis, which is where individual learning, group becoming and community practices are often most visibly displayed (Stahl, 2015c).

Findings: Collaborative Learning of Dynamic Geometry Core Principles

In our case study for this presentation, three 14-year-old girls engaged in our environment for eight hour-long sessions (Stahl, 2015a). In their sixth session, they worked on the problem shown in Figure 1, constructing inscribed squares. They had previously solved the challenge of constructing inscribed triangles, but had never constructed a square. We follow their explorations, which led to an elegant construction of a square. They were then able quickly and collaboratively to construct the inscribed squares, based on their previous experience with inscribed triangles. They displayed their group and individual learning through their GeoGebra actions, text chat and building on each other. They explicitly discussed the need to construct various geometric dependencies to accomplish this task.
Conclusions: Designing an Integration of Software, Curriculum and Practices

The Virtual Math Teams (VMT) Project (http://gerrystahl.net/vmt) at the Math Forum (www.MathForum.org) has been researching the integration of: an online collaboration environment, multi-user versions of GeoGebra, sequences of curricular units, data analysis methods and pedagogical approaches for over a decade. We now believe that a collaborative approach to dynamic geometry can support the learning of core components of mathematical cognition. Our approach integrates online software (for all browsers on computers, tablets, iPad), student-centered collaboration (with text chat), teacher orchestration of student teams and a carefully scripted sequence of curricular units (emphasizing exploration, reflection and group mathematical discourse). The curriculum is aligned with Common Core standards and focuses on the mathematical notion of dependency and techniques for constructing dependency in GeoGebra. Dependency is central to dynamic geometry, to deductive thinking and to student understanding of explanatory proofs. The dependencies constructed with GeoGebra tools—often following Euclid’s procedures—result in figures with desired invariants. We have shown that even young students in groups can begin to understand, analyze, design and construct dynamic-geometric dependencies with GeoGebra.

The tablet version of multi-user GeoGebra with chat has just become available (vmtdev.mathforum.org). Teachers and groups of students can use it for free. A set of 50 GeoGebra activities (in a GeoGebraBook: http://ggbtu.be/b140867) introduces student teams to the role of geometric dependencies in exploring, articulating, creating and explaining dynamic-geometry figures and relationships—within a gaming-like context of sequenced challenges.

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USING OFF-THE-SHELF TOOLS TO SUPPORT MATHEMATICS COLLABORATION:
TECHNICAL AND HUMAN SUCCESSES AND ISSUES

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Abstract

GeoGebra, along with other web-based resources such as wikis, virtual walls, and screen casting can be combined to provide online collaboration spaces to support student sharing and discussion of mathematical ideas as called for by teacher associations (NCTN, 2000; OAME, 2011) and curriculum guidelines (Ontario Ministry of Education, 2005, 2007). This paper and the related presentation will outline our design experiments combining these online tools and the related outcomes. Although we have been able to address technical issues to construct fully functioning online collaboration environments our classroom implementation has not been as successful as we imagined. We have met human issues related to the images of mathematics held by students and conflicts between our proposed use of the web and students' experience with social media. We will outline these issues and our efforts to address them.

Nason and Woodruff (2004) argue that online collaborative learning environments do not support the simple exchange of mathematical ideas in symbol or image forms. More recently Mayer (2012), writing about the difficulties involved in mathematical communication, makes a similar claim; stating that “at this moment in history, computers are not a natural working medium for mathematics”. Our classroom design experiments (names withheld for blind review, 2013a) contradict such views and show that in fact students can communicate about and share mathematical ideas on line. In addition, this online sharing could be carried into other classrooms since the collaboration environments have been constructed using open-source tools and software and services provided free by commercial operations as trial versions of their “pro” packages. We demonstrate that technology does exist to support students engaged in online mathematics collective problem solving. But, at the same time our work has revealed complicating issues on the human side; concerning the popular images of mathematics and the readiness of the net generation to engage in online activities that go beyond their experience with social media.

Mathematics sharing environments have been constructed and employed with Ontario secondary school students in grades 10 and 12, by combining wikis (PBworks, http://pbworks.com), Padlet
(http://padlet.com/), GeoGebra (International GeoGebra Institute, www.geogebra.org/), and Jing (http://www.techsmith.com/jing.html). The wiki or Padlet provides a place for the teacher to present a task or problem and for students to upload their contributions to conversations working towards a solution. Students present their mathematics in GeoGebra applets embedded in or linked to the wiki or Padlet entries. Explanations of the steps and thinking are provided in text or more extensively in the form of Jing screencasts created while contributors use GeoGebra to explore the mathematics. This environment readily supports a building-on process. By clicking on a posted GeoGebra applet a student gets a live version of the software in the form left by the previous contributor. They can then pick up the partially constructed solution, complete additional mathematical manipulations within GeoGebra, and then submit a new contribution that advances the group’s thinking.

Although working with each of the individual tools employed in our collaboration environment is not overly difficult the total package can be confusing. Thus we have employed a very deliberate step-by-step approach to introducing online collaboration and building the required skills. Initially the teacher needs to spend time building a supportive social environment in the classroom and developing the idea of collaboration in mathematics. On the technology side the initial steps involve using a wiki or Padlet as a place for the teacher to present course material. Once students are familiar with locating material online they move on to using text to submit questions concerning course work and solutions to problems. Parallel to this, GeoGebra skills are developed through offline use of the software. Once there is sufficient familiarity with the wiki and GeoGebra students share constructions by uploading their work to GeoGebraTube (https://www.geogebratube.org/) and linking it to the wiki or Padlet. Extended text-based descriptions of mathematical activity and thinking are somewhat tedious to write and so students are introduced to the process of recording their computer screen and voice while working with GeoGebra. These videos, holding contributors’ thinking, are uploaded to a Screencast library (http://www.screencast.com/) and embedded in or linked to the wiki or Padlet along with the associated GeoGebra applets.

The collaborative learning in which we wish to engage students stems from the view that mathematics is a human construct developed over centuries as people collectively built a language to describe the natural and human-built world (Ernest, 1998). This image of mathematics is reflected in the concept of the classroom as a math-talk learning community (Hufferd-Ackles, Fuson & Gamoran-Sherin, 2004); a view that has been promoted by the Ontario Ministry of Education (Bruce, 2007; OME, 2008a, 2008b). Despite curriculum policy support for a rich image of mathematics, most students appear to hold a tool-kit conception of the discipline; one that pictures the subject as a set of fixed rules and procedures that must be accepted as fact and memorized.
With this view mathematical discussions and sharing make little sense. Since success in mathematics is defined by getting the correct answer to teacher or textbook posed problems, students are reluctant to share their conjectures and partially formed solutions. Thus success in building student collaboration, face-to-face or online, requires initial teacher efforts to help students build a richer image of mathematics.

We have found that the popular view of today's youth as members of the "net generation"; possessing a wide range of digital skills (Tapscott, 2009), is not universally valid. In the classrooms where we have worked there was a wide range in digital experience and also a range in enthusiasm for online activities. Among those students engaged in online activities, experience was largely confined to texting between friends, reading and posting to Facebook pages, and viewing YouTube videos. The students with whom we work are not unique. Canadian data related to online experience (ComScore, 2013, 2014) show that social networking and watching videos are the dominant activities, particularly for the 15-24 years age group (ComScore, 2011) that includes secondary school students. The skills developed in these areas are not directly transferable to working with GeoGebra and being the producer rather than just consumer of online videos.

Students' social media experience also leads to particular expectations concerning the look of online spaces and the ease of uploading content. When communicating in text, youth tend to employ a short message service and use a very economical code. Such messaging experience is not compatible with the more extensive communication required for explanations of mathematical processes. This conflict with the present experience of young people was the motivation to introduce the use of screen capture videos and have our students communicate in an environment more similar to YouTube (names withheld for blind review, 2013b). As teachers we appreciated the structure provided by a wiki, but the students appeared to find this restrictive. They are familiar with the simple click and type or upload approach of Facebook. Our recent adoption of Padlet as the organizing space is a partial move in the Facebook direction.

We are convinced that free or inexpensive technologies exist to support online collaboration in mathematics. The rapidly expanding digital world gives increasing opportunities in this direction, but also presents us with issues to be addressed.

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THE BENEFITS OF STUDENT-CREATED GEOGEBRA APPLETS IN A PRECALCULUS COURSE

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Abstract
I would like to present two types of GeoGebra applets that I have my Pre-Calculus students create, to present samples of their actual work, and discuss the benefits from both my point of view and theirs, as taken from their own reflective writing on these tasks.
I have found that even though they struggle with these tasks, most of the students persist far more than they do with more traditional assignments. This type of struggle does not seem to defeat them, rather, for the most part, it motivates them to keep trying until they are successful. The amount of time spent at the beginning of the year teaching them how to create these applets is more than made up for with engagement and depth of learning.  

Motivation
I used to create GeoGebra applets for my students, which they would use to answer my questions. I still do this, but I also get them to create their own GeoGebra applets. The reasons for this are numbered here so that I can refer to them in the task description following this section:

1. I worried about my students becoming too reliant on the dynamic worksheets that I created, because their algebra skills weren’t being developed if they could find the answer to “What is the x-intercept of this function?” by moving sliders instead of working it out algebraically.

2. When it came time for a paper test, I had to of course deny them the use of GeoGebra and require them to show all their work, which seemed a bit contrived and artificial. If GeoGebra is so valuable, then why would it not count as part of their assessment? And, if it’s so easy to find the answer using GeoGebra, why would anyone have to be able to do it on paper? (Comparable to the argument “Why learn to code when you can just turn on the computer and it works just fine?”)

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3. One of the greatest benefits of GeoGebra was not being realized - using it to ask and answer their own questions. It's a tool that can create other tools to keep the learning moving forward, but when I was the one creating the applet, once they'd answered my questions, they wouldn’t look at it again.

4. Learning should be unpredictable, and I had a strong intuition that the potential for learning unexpected things using GeoGebra would increase dramatically, for my students and for me.

**Description of Innovative Practices: Explorers and Virtual Manipulatives**

**Explorers**

For each function that we study, my students create a GeoGebra applet in several versions, each more complex than the last. The first contains the 4 sliders to control all four function parameters. This alone requires careful input of the parameter-laden equation, with brackets in the right places, so that the a, b, h, and k sliders do what they’re supposed to do. Verifying that it’s working properly requires careful inspection, as well as comparisons to their own graphs done on graph paper, all of which are documented in the students’ digital portfolios (examples of which will be shown during presentation.) This addresses motivation points 1 and 2, the concern for developing algebra skills and motivating manually working out examples on paper.

The next version has one extra slider - a t-slider, representing a time variable. As the t-slider is moved, it must cause a point to slide along the function’s curve, the x-coordinate of which is the current value of t on the slider. (This is partly in preparation for the projectile projects described below.) This means they have to create a point that contains a formula involving the function rule and t. For the linear function, it would look like (t, mt + b). This reinforces the concept of domain, for example, for the square root function, the point (t, √t) won’t show up on the curve whenever t < 0. Some students become wonderfully frustrated when this happens, thinking they’ve made a mistake, or that “GeoGebra’s not working”, until the moment they realize the reason it’s not showing up is that it doesn’t exist. This is the most effective demonstration of the meaning of domain that I’ve ever tried. This version also helps develop their algebraic sophistication in that they see the x and y variables as a different kind than the parameters, since as they move the t-slider, and x changes, the parameters are constant. This addresses motivation point 1 - in fact it goes far beyond, in that it created an opportunity to develop algebra skills that I hadn’t previously even thought of. Some unexpected bonuses, learning-wise and math-wise, came about when one student discovered how to turn on the trace feature and animate the t-slider. She learned this by

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searching and finding the website of one of my own GeoGebra mentors, Steve Phelps. Not only did she and I thus become part of each other’s PLN (Personal Learning Network), but when her GeoGebra started to trace a parabola from a point moving on a straight line, I learned something about lines and parabolas! This addresses motivation point 4, creating the potential for the unexpected, for everyone.

The next version displays the important points of the function, like the vertex and intercepts. For example, for the function \( f(x) = mx + b \), in order for the \( x \)-intercept to be in the correct place, regardless of the current values of the parameters, they need to input a formula for the coordinates of the zero, such as \((-b/m, 0)\), or \((-b ÷ m, 0)\). This formula will have to use, instead of numbers, the letters representing the parameters. This is likely the first time they have had to create a formula, as opposed to using one that someone else worked out and handed to them. At first, for most students, this requires examples to be worked out on paper with actual numbers, in order for the students to detect the pattern, which then leads to a formula. Once they have one, they can find out immediately if it’s correct simply by looking at their own applet— if it’s always in the right place, no matter how much they move the sliders around, they need no assurance from me, or anyone for that matter, that their formula is right. As the year progresses, we fall naturally into manipulating equations in which there are no numbers, but instead parameters and variables, a very valuable skill for anyone pursuing higher STEM education. Another benefit in terms of algebraic sophistication is that two students’ formulas might look different even though they both work. This is an opportunity to motivate the reason for simplifying algebraic expressions, to show that they are in fact equivalent, not to mention making a case for simplicity and eloquence in one’s formulas. This addresses motivation points 1 & 2, the development of algebraic skills, in fact, it more than compensates for whatever reliance they may have had on my GeoGebra applets.

Subsequent versions display other properties, such as domain and range, intervals of increase or decrease, asymptotes, etc., and as the year goes on, some students even use conditional features, such as changing the colour of the curve when it’s increasing, or having a text message pop up that says “This function has no \( y \)-intercept.” Their final version is a complete study of the current function and all its properties. This used to be what I would create and hand to them early on in our unit, inviting them to use it to check their paper work. Now they build it slowly during the course of the unit, and by the time they’re finished, not only have they done the paper work and the algebra, but they’ve gained a deeper understanding of the concepts along the way. As we move through the year, they apply their GeoGebra skills to create the next function explorer, and use the old ones to contrast and compare the different function families, which addresses point 3, that their own creations move their learning forward continuously.
Virtual Manipulatives

I collaborate with the Physics teachers at my school on a project about projectiles. The students use GeoGebra to create a virtual projectile whose path is controlled by 4 sliders: launch angle, launch velocity, initial x-coordinate, and initial y-coordinate. In order to make the projectile move along its path, they must also have a t-slider whose movement causes the projectile to move. One benefit that we see is the students’ increased comfort with the formulas for the horizontal and vertical positions of the projectile that they learn in Physics. For most, it is their first exposure to parametric equations. Even though by the time they are creating their GeoGebra projectiles, they have already used these formulas to do many calculations, seeing them actually cause their projectile to follow a parabolic path is eye-opening for them. The project is also a great opportunity to let students’ creativity take wing. They can make it about any type of projectile they want, and they can be as creative as they want in terms of making a background to enhance their display. Furthermore, once their projects are completed, they continue to use them to study projectile motion in more depth in their physics class, which addresses motivation point 3 – that the GeoGebra applet they create should continue to be used to move their learning forward.

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GAZE COORDINATION DURING COLLABORATIVE PROBLEM SOLVING IN GEOGEBRA

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Introduction

The dual eye tracking paradigm, where the eye gaze of multiple subjects are monitored simultaneously while they are collaborating on a shared task, provides important opportunities to investigate the mechanisms underlying joint attention and shared understanding in the context of collaborative learning (Nüssli & Jermann, 2012). The degree of overlap or cross-recurrence among gaze movements of participants provides important information regarding to what extent the participants mutually orient to each other and to the objects in the shared scene (Richardson, Dale & Kirkham, 2007). A general finding in this emerging literature is that there is a positive correlation between the degree of gaze recurrence and collaborative learning outcomes (Jermann et al., 2011). These findings also motivated the design of learning environments where users’ gaze information is visualized on the screen in real-time as an awareness mechanism. A recent study on such a joint learning environment reported significant gains in gaze coordination and learning outcomes (Schneider & Pea, 2013). With the emergence of affordable eye tracking systems such as EyeTribe and Tobii EyeX, integrating such systems in collaborative environments to support joint attention seems to be an increasing possibility.

In this research paper proposal we focus on the relationship between changes in gaze recurrence and the sequential organization of interaction in a chat environment called Virtual Math Teams (VMT), where we asked student dyads to collaboratively work on dynamic geometry problems. In this short paper we present our preliminary findings regarding how measures of gaze recurrence relate to the overall success of collaboration in the context of collaborative math problem solving online.

Methodology

The study was conducted with 18 volunteered university students at METU in Ankara, Turkey. Participants were grouped into 9 pairs and asked to collaboratively work on 10 geometry problems by using the VMT environment (Stahl, 2009) (Figure 1). Before the experiments participants received a short training that illustrated basic features of VMT. The tasks were selected among activities
presented in Stahl (2013), which included tasks such as constructing a perfect equilateral triangle, an isosceles triangle, or a perfect hexagon in GeoGebra. Participants coordinated their work and discussed strategies via the chat, and took turns to construct the desired dynamic geometry representations.

During the online sessions eye movements of the participants were recorded with two desktop eye trackers (Tobii T1750 and Tobii T120) at a resolution of 50Hz. Gaze recordings were split into synchronized excerpts in the Tobii Studio Software. The screen recording of each participant was then split into a 4x4 matrix of non-overlapping areas of interests (AOIs) as shown in Figure 1. 12 of the AOIs covered the area where the dynamic geometry representations were constructed and 4 AOIs covered the chat component on the right. Each of these 16 regions was considered as an approximation of the part of the screen over which the participants were attending to at any given time.

While monitoring the shared scene users either move their eye gaze with saccadic movements or fixate on specific locations by keeping their eyes still over a location. During a fixation event the fovea, which is the part of the retina that has the highest concentration of light sensitive cells, is oriented towards the fixated location. The fovea covers approximately 1-2 degrees of visual field, and at a distance of 65 cm from a screen, 1-2 degrees of visual field corresponds to a circular area with a diameter of 2.2 cm on the screen (Duchowski, 2007). The visual attention span is considered to cover a larger area covered by the foveal projection, as evidenced in dual-task experiments (Holmqvist et al., 2011). Since 17 inch displays with 4:3 aspect ratios were used during our experiments, the width and the length of the screen was 35 cm and 26 cm respectively. Splitting this area into 16 equal non-overlapping rectangular AOIs covers an area approximately 9 cm wide and 7 cm long. In this study this rectangle was considered as a rough approximation of where the person is attending to at any given time.

![Figure 1. A screen shot of the VMT environment is presented on the left. The screen shot on the right shows the 16 areas used for partitioning the screens into areas of interest.](image-url)
Using the same AOI definitions on both screens over synchronized eye-tracking data allowed us to quantify gaze overlaps. Since the screens were divided equally, the probability that one of the participants allocate their attention on a given AOI is 1/16. Assuming independence of random gaze events, the possibility that two people allocate their attention on the same AOI is 1/16x1/16 = 1/256. So, the likelihood of having systematic gaze overlap among 2 people by chance is 0.004, despite the low spatial resolution provided by 16 AOIs.

A gaze recurrence plot was produced for each problem attempted by every pair. The plots computed for each pair were then combined into a single recurrence percentage plot by taking the average of all corresponding data points coming from the plots for individual problem solving segments (see Figure 2). This yields a recurrence plot that summarizes the gaze patterns over all problem solving segments of the particular pair. In the combined summary plot, data points range from -4000 msec to +4000 msec with a 100 msec resolution. Point 0 indicates the recurrence percentage for synchronous gaze overlaps, -200 indicates the recurrence percentage in which B’s gaze follows A’s with a 200 msec delay with respect to A, and vice versa. The blue line shows the baseline recurrence level, which is computed over randomly shuffled gaze data. The vertical lines are the standard error bars, which indicate the amount of deviation in the data for the corresponding time.

Results & Discussion

In order to test the relationship between group performance and the degree of gaze overlap, 9 pairs were split into 3 achievement groups in terms of the number of problems they could solve correctly, whether they worked on tasks in a coordinated way and whether both participants contributed to the discussion. The degree of gaze overlap observed during each session was used as an indicator of the level of joint attention achieved by each group. Previous studies conducted with voice enabled computer-mediated communication systems found that participants took on average approximately 2 seconds to focus their attention on an object after it was mentioned by her partner (Richardson & Dale, 2005). In the present study the communication among partners is mediated by a chat and a shared drawing tool. One may expect that reading a chat utterance and then allocating one’s attention to the referred object on the drawing board take more than two seconds. However, the gaze recurrence plots with various lag combinations indicated that the highest degree of gaze overlap occurs within a similar time interval. Therefore, those instances in which one subject looks at the same area of the screen that his partner looked at within two seconds were treated as instances of gaze overlap.

The high achievement group exhibited on average 31% gaze overlap, which is followed by the medium and low achievement groups with 24% and 13% gaze overlap respectively. A one-way
ANOVA conducted over gaze overlap values indicated that this difference is statistically significant, $F(2,8) = 11.917$, $p<0.001$, $\eta^2 = 0.341$. Games-Howell post hoc tests found a significant difference between low and medium achievement groups (MD=12.32, $p<0.05$), as well as between low and high achievement groups (MD=20.19, $p<0.01$). The difference between medium and high achievement groups was not significant. In other words, higher achieving groups exhibited significantly higher gaze coordination during collaborative problem solving sessions.

Figure 2 shows the overall gaze recurrence plots corresponding to the entire session of two different dyads. The plot on the left of Figure 2 corresponds to the best performing dyad in our sample, whereas the plot on the right corresponds to the worst performing team. The better performing team’s recurrence plot differs from the other team’s plot in several ways. First, the mean percent recurrence values are much higher for the better team. Second, the better team’s recurrence plot significantly deviates from the random baseline, whereas the other team’s recurrence plot cannot be distinguished from the baseline. The recurrence plot for the better team has two peaks, one around -1500 msec, whereas the other at 1500 msec. This suggests that both partners equally followed each other’s gaze, indicating a high degree of coordinated behavior. Such a pattern is not visible for the other team.

These quantitative results indicate that the degree of gaze recurrence could be a good predictor of effective collaboration in the context of collaborative math problem solving with dynamic geometry representations. Gaze recurrence analysis provides a global view of gaze patterns, where one can see if there is a significant amount of gaze coordination and if there is an asymmetry among peers (e.g. one partner tended to follow the eye gaze of the other). For instance, the cross recurrence map for the successful dyad was symmetric with respect to the origin, meaning that both partners almost equally followed each other’s gaze, which signals that there is a stronger level of coordination among their actions.

In our ongoing work we are qualitatively investigating how variation in gaze recurrence relate to specific actions on dynamic geometry representations. One particular type of action that differed the more successful team from the other was the presence of anticipatory gaze overlaps, where the student who is watching his/her partner’s construction fixated on the location where the next relevant step of the construction was expected to occur. Moreover, dragging actions seemed to generate a powerful visual effect on the shared space, which effectively elicits the other partner’s attention in the form of a smooth pursuit gaze event. We are in the process of building a taxonomy of such gaze overlap patterns relevant for characterizing the effectiveness of collaboration mediated by GeoGebra representations.
Figure 2. Overall gaze recurrence plots for the highest achieving and the lowest achieving dyads.

References


WORKING GROUPS
WORKING GROUP 1:
DEVELOPING COMPREHENSIVE
OPEN-SOURCE GEOMETRY CURRICULA USING GEOGEBRA

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Abstract
Imagine combining the best characteristics of your favorite new geometry textbook with GeoGebraTube. It would cover all the material required for your ideal version of a full course on geometry, easily accessible and usable by teachers and students. However, it would also be free, flexible, up-to-date, easily revised and downloadable as needed by teachers and students. It would include activities tested in diverse classrooms, reviewed by teachers and flexibly adaptable to different languages, cultures or pedagogical preferences. Perhaps most importantly, it would take full advantage of GeoGebra for a dynamic, hands-on, visual, drag-able, constructible, personalizable exploratory-learning approach to geometry. The currently missing piece for moving GeoGebra into the center of contemporary mathematics education is the availability of comprehensive curriculum aligned with standards. GeoGebraTube provides a medium for shared resources, but requires coordinated efforts to develop model curricula and an interface for flexible, collaborative usage.


Keywords. Curriculum Development, Collaborative Learning, Common Core, GeoGebraTube, GeoGebraBook, Open Source, Search and Browsing Software.

Working Group Goal
The goal of this working group is to stimulate development of comprehensive geometry curriculum centered on student use of GeoGebra. This will support the use of GeoGebra by geometry teachers around the world by helping them to integrate student use of GeoGebra into their classroom activities, enhancing the pedagogy. This working group is only intended to start the process. Perhaps it will stimulate people thinking seriously and strategically about possible approaches and put them in contact to pursue next steps. Success with basic geometry could provide a model for other areas of mathematics.
Problem Statement

While some new textbooks and the US Common Core standards recommend use of dynamic-geometry environments to “provide students with experimental and modeling tools that allow them to investigate geometric phenomena,” they put the burden on the teacher of realizing this in the classroom. However, curriculum development and the construction of the corresponding well-designed GeoGebra files is a sophisticated and time-consuming task. Teachers have neither the time nor the resources to do this on their own for a whole course on geometry. They need well worked out curricula that they can choose from and adapt to their local needs.

Curricular items are currently made available through GeoGebraTube. However, that software does not support the assemblage of comprehensive, well-organized and easily adapted curricula. Nor does it support collaborative usage by student teams. GeoGebraBook can be a first step, but more is needed.

Curriculum in GeoGebraTube is currently unorganized; it is not systematic or comprehensive; it is not tied to progressive pedagogies. The consequence of this is a serious under-utilization of the potential of GeoGebra in typical classrooms. Without well-tested tutorials and curricula for important topics like construction, proof or custom-tool programming, teachers tend to fall back on using GeoGebra for fancy visualizations, and students use it to create pretty pictures. The power of dynamic geometry to stimulate mathematical thinking and cognitive development of students is barely touched.

Working Group Focus

This working group will focus on enumerating the major issues and the main tasks that need to be addressed initially. The central question is how to support the integration of GeoGebra into geometry courses around the world. This includes approaches to both collaborative learning in small groups and individual learning. A particular opportunity of the Internet-based single-user and multi-user versions of GeoGebra is their use by online schools and for networking home-schooled students or students in countries with dispersed populations. Although intended to be useful for students world-wide, the curriculum might be aligned with the US Common Core standards as a framework. Although it is not necessary that GeoGebra be used for every aspect of school geometry or other math courses, the target curriculum should support a strategic, systematic approach to the aspects that it does address.

Background Information

The Virtual Math Teams (VMT) Project (2003-2014) developed an online environment for collaborative dynamic geometry using a multi-user version of GeoGebra and text chat (Stahl, 2006;
It developed an associated mini-curriculum focused on collaborative learning of construction of geometric dependencies. This curriculum has been tested and revised each year. A version is now available as a GeoGebraBook (Stahl, 2015); it focuses on developing an understanding of how to construct geometric dependencies based on the beginning of Euclid’s Elements and explores many notions recommended by the Common Core for middle school. This active book lets students work on 50 individual challenges in GeoGebra. Unfortunately, it is not multi-user, it is not persistent, there is no chat and it is not instrumented for researcher analysis, student learning analytics or teacher supervision.

An earlier project, the Teachers Curriculum Assistant (TCA) designed in 1994, explored the possibility of searching and browsing a database of curricular materials even before the Web existed (Stahl, 2006, Ch.1; Stahl, Sumner & Owen, 1995). It focused on five principles for a shared repository of constructivist educational resources, which could be applied to GeoGebraTube as follows:

1. Carefully structured summaries (meta-data) of the resources must be defined (when they are uploaded) and maintained, to support search. (GeoGebraTube begins to do this.)
2. The search process should be supported through a combination of query and browsing tools that help teachers explore what is available. (GeoGebraTube provides a simple search.)
3. Adaptation of tools and resources to teachers and students is critical for developing and benefiting from constructivist curriculum. (GeoGebraTube allows editing, but not versioning.)
4. Resources must be organized into carefully designed curricular units to provide developmental learning sequences. (GeoGebraTube has tags and Books, which are a start for this.)
5. GeoGebraTube should be a medium for sharing and combining curriculum ideas, not just accessing them. (In GeoGebraTube, “sharing” is just sending a link through social media.)

The following components of TCA were designed: a Profiler, Explorer and Versions (see figure below on the left) as well as a Planner, Editor and Networker (on the right). They are suggestive of useful functionality. These allow curriculum developers and teachers around the world to search for resources, try them, edit, annotate and store new versions. They also facilitate the aggregation and structuring of coherent sequences of curricular resources into course modules adapted to local needs of specific countries, schools, teachers or students.

Individual curricular resources and coherent sequences of them can be linked to relevant pedagogical materials, such as instructional models, logs of student usage, experience reports, pedagogical recommendations, ratings and reviews. This supports a broader conception of curriculum, including examples of pedagogical approaches, models of successful usage by
teachers and students, discussion of alternative options, assessment instruments and research analyses.

**A MOOC Model for Collaborative GeoGebra**

Massive Open Online Courses (MOOCs) and sites like Khan Academy provide useful educational resources, but they generally involve passive watching of video lectures, rather than engaged social learning. The VMT approach suggests a collaborative model, integrated with local classrooms and teachers. GeoGebra Institutes can provide teacher professional development in the proposed curricula. Then teachers can adapt the curriculum to integrate with their courses. Teachers organize small groups of their students to work collaboratively on GeoGebra curriculum, motivating each session in advance, then sharing group findings in whole-class discussions. The teachers guide the exploratory-learning trajectory and manage the grading (with automated support from the software). This overcomes the problems of MOOCS, takes advantage of large-scale resources and supports local mathematics education.

**Discussion Structure**

The author team will begin by (a) motivating and illustrating the topic with the example of the VMT Project, its pedagogical approach to exploratory collaborative learning, and its sample GeoGebraBook curriculum. It will then (b) facilitate open discussion, starting with the questions and topics listed below. Finally, there will be (c) a wrap-up enumerating priorities, next steps and potential participants.

- How can comprehensive curriculum advance teaching and learning with GeoGebra?
- What new features should be designed into GeoGebraTube and GeoGebraBook to support meta-data, searching, browsing, adapting, annotating, reviewing, linking etc.?
- How should GeoGebra Institutes be involved? Should there be a form of MOOCS?
- Can curriculum be designed to support and assess both collaborative learning and individual learning?
- How can teachers be supported to adapt curricular units to their classrooms and how can they be involved in evolution of the materials?
- How can examples of teacher approaches, student work, assessment instruments, etc. be integrated into the materials?
- What resources are currently available and what further resources—such as research funding—should be sought?
- Who is interested in collaborating in further work on this?

References


WORKING GROUP 2:
WHAT ARE THE AFFORDANCES OF GEOGEBRA AND OTHER DYNAMIC LEARNING ENVIRONMENTS FOR TEACHING AND LEARNING OF PROBABILITY?

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Abstract
The working group focused on the use of GeoGebra (and other dynamic visualization software) learning and teaching probability. We addressed several issues in the discussion such as the importance of learning probability, frameworks describing criteria for understanding probability, technology use in teaching and learning probability in conjunction with the visual characteristics of new learning form of probability. Among various representations used in mathematics and mathematics education, visual representations of probability concepts, the effects of the implementation of visual techniques, and more importantly the importance of visual exploration of probability and its contribution to learning probability were discussed. The discussants discussed the merits of traditional Venn diagrams and explored the construction of various representations of probability events.

Background
The focus of this discussion group is situated at the intersection of the technology use in mathematics education, visual learning in mathematics education, and developing a conceptual understanding of probability. An enormous corpus of literature has been accumulated on technology use in mathematics education and visual learning of mathematics (Arcavi, 1999; Duval, 1999; Kaput, 1999; Kaput & Hegedus, 2000; Leatham, & McGehee, 2004; McDougall, 1999; Moreno-Armella, 1999; Presmeg, 1999; Thompson, 1999).

Probability Learning Frameworks
There are several frameworks and building blocks (“lists of criteria”) that describe probabilistic reasoning. The major frameworks mentioned in the literature are: core domain of probability concepts (Moore, 1990), probability thinking framework (Jones et al., 1997), and building blocks of probability literacy (Gal, 2005). These frameworks can be divided into two groups: prescriptive and descriptive.

Moore’s (1990) probability thinking framework is primarily prescriptive because it describes the pieces of knowledge that a person needs to have in order to be able to reason probabilistically.
Moore (1990) describes the conditions that students need to satisfy in order to be able to move towards more difficult concepts including conditional probability. These conditions are: (1) learning to discern the overall pattern of events and not attempt a causal explanation of each outcome; (2) recognizing the stability of long run frequencies; (3) assigning probabilities to finite sets of outcomes and compare observed proportions to these probabilities; (4) overcoming the tendency to believe that the regularity described by probability applies to short sequences of random outcomes; and (5) applying an understanding of proportions to construct a math model of probability and develop an understanding of some “basic laws or axioms that include the addition rules for disjoint sets.” (p. 120). The first condition deals with the understanding of randomness and is relevant to this thesis.

Gal’s (2005) model consists of two parts which the author calls “the building blocks.” (p. 46). The first set of “building blocks” consists of the knowledge elements. The knowledge elements are prescriptive because, similar to the Moore’s model, it lists pieces of knowledge that students should have in order to master probability. The second part of the model consists of dispositions that play “a key role in how people think about probabilistic information or act in situations that involve chance and uncertainty, whether in the real world or in the classroom” (Gal, 2005, p. 45). This part of the model is descriptive because it does not exclude dispositions that are detrimental for understanding probability. In Gal’s model, the knowledge elements are: (1) big ideas (variation, randomness, independence, predictability, and uncertainty); (2) figuring probabilities (ways to find or estimate the probability of events); (3) language (the terms and methods used to communicate about chance); (4) context (understanding the role and implications of probabilistic issues and messages in various contexts and in personal and public discourse); and (5) critical questions (issues to reflect upon when dealing with probabilities). Dispositional elements in Gal’s model are: (1) critical stance; (2) beliefs and attitudes; and (3) personal sentiments regarding uncertainty and risk (e.g., risk aversion).

All above-mentioned frameworks imply the following criteria for understanding of probability: (1) acknowledgment that not all phenomena can be viewed causally and deterministically; (2) understanding that random events might have a pattern that is stable in the long run, but that this pattern cannot be used to predict the next outcome; and (3) acknowledgment that randomness is a concept that permeates all aspect of life.

**The Rationale and Goals of the Discussion Group**

The goals of this discussion group are to explore and discuss the technological and visual approaches in learning and teaching probability, to improve awareness on the technology use in
conceptual understanding of probabilistic concepts, and to set up a research agenda on the study of technology use in learning probability.

We also expected to review theoretical discussions in learning probability, the past and current use of technology in learning probability and seek for opportunities for possible use of technology in the future. Another objective was to seek possible ways to deepen our understanding of our basic concerns in understanding and teaching probability and the contribution of technology to cope with these concerns. In addition, we expected to find opportunities to implement open source technology in classrooms to learn and teach probability.

Summary of the Working Group Activity

In the first part of the session, Nenad Radakovic discussed the challenges of teaching probability in the secondary setting, particularly the problem of teaching Bayesian inference. He presented Area Proportional Venn Diagrams and the algorithm of constructing them in GeoGebra. The geometric interpretation of the area-proportional Venn diagrams was presented (Figure 1). The discussants then suggested various ways of representing Bayesian problems and the alternatives to using circles to represent sets (events). Gerry Stahl presented a simpler construction using squares rather than circles.

![Figure 1. Geometric representation of 2-set area-proportional Venn diagrams](image)

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WORKSHOPS
WORKSHOP 1:
CONSTRUCTING AND SHARING AN ONLINE GEOGEBRA LEARNING OBJECT

Geoffrey Roulet
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Abstract
GeoGebra; being coded in both Java and HTML5 forms, can be run on multiple computer and tablet platforms and integrates well with the web. Thus it is an ideal tool for teachers who wish to use online spaces to share resources with their classes. Many such resources have been created by GeoGebra users and are shared via GeoGebraTube (http://tube.geogebra.org/). You might find the learning objects you need there, but if not, GeoGebra provides tools that should make it possible for you to construct any resource you require for your courses. This workshop will lead you through the steps in creating and sharing a learning object. We will construct one particular resource, but the process should provide a template for you to use to build other learning objects for your class.

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WORKSHOP 2:
DEVELOPING PATTERNS, ORNAMENTS AND REVISITING TRANSFORMATIONAL GEOMETRY WITH GEOGEBRA

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Abstract
The aim of this paper is to provide documents for the workshop conducted at the fifth North American GeoGebra Conference, November 22, 2014. The workshop falls somewhere between Computer Technologies for learning/doing mathematics and curriculum/software development, mostly focuses on the ways using computers in learning and doing mathematics through an explorative lens. The theoretical framework of the proposal falls in the Realistic Mathematics Education. That is, using mathematics and technology to understand the world around us and using mathematics and technology to improve our cognitive abilities. The workshop is planned as to introduce the basic features of GeoGebra related to transformational geometry.

GeoGebra in learning transformational geometry
Many students consider geometry as a collection of figures, such as circles, squares, triangles, etc., and some properties associated to these figures. Geometry, for those students, is list of definitions of aforementioned geometry concepts and the formulas connected to these definitions. They usually attempt to memorize the rules without actually understanding of their connections to the real world. Even students perform well in geometry courses do not have enough experience on the geometry situated in real world. For some cultures, it is actually a waste of opportunity because each culture has some degree of interaction, my personal assumption, with geometry.

This paper is a working paper for an attempt to unfold the geometry situated in artefacts around us and to re-connect this geometry to Euclidian geometry by employing technology. However, only unfolding the geometry situated in cultural artefacts, Islamic Art in particular, and then, to recreate one unit of pattern in the GeoGebra will be addressed in this workshop. The workshop will start by reviewing the basic features of GeoGebra and transformational geometry to refresh participants' knowledge. The procedure and details for this could be found in Appendix A.
First, a unit cell of a triangular pattern will be created by employing reflection and rotation. Second, the same construction will be created dynamically, using a slider. Third, a generalized version of the construction will be created. Finally, the dynamic construction will be created by using GeoGebra commands.

Then, in the following section, some examples of geometry situated in Islamic Art will be visited. Further work on how to create those geometry constructs will be discussed in the later publication.

The theoretical perspective of this workshop falls into the Realistic Mathematics Education (RME), first was put forward by Freudenthal (1971, 1973), and then elaborated by others, such as Treffers (1987) and Gravemeijer et al. (2000).

**Supporting Materials**

The supporting materials for the workshop will also be available on the Conference and Institute website.

- The Fifth North American GeoGebra Conference website (https://sites.google.com/site/geogebranorthamerica2014/ )
- GeoGebra Institute of Canada website (http://www.geogebracanada.org/)
Appendix A: Transformations

To draw a 30-60-90 degree triangle (Figure 1)

- Create the points A and B,
- Create a line passing through these points,
- In order to create an angle of 60 degree at the point B, first activate the “Angle with Given Size” icon and then click on the points A and B, respectively. Do not forget to select the “clockwise” after typing 60 degree in the dialog box,
- Draw a line passing through the points A’ and B,
- Draw a line perpendicular to the line AB at the point A,
- Active the “Intersect Two Objects” icon and select the perpendicular line and the line making 60 degree with the horizontal line.

Figure 3. A construction of 30-60-90 degree triangle.
To reflect the triangle over its hypotenuse (Figure 2)

- Make the lines AC and AB, the point A', and the angle α invisible by right-clicking on each and select “Show Object,“
- Select the “Polygon” icon and then click on the points A, B, C and again A to create ABC triangle,
- In order to reflect the triangle ABC about the line BC, activate the “Reflect Object About Line” icon and then click on the triangle ABC and the line BC, respectively.

Figure 4. A kite obtained through reflection.
To rotate the kite around a point of reflection (Figure 3)

- Create a point below the kite and name it as PofR (Point of Rotation),
- Activate the “Rotate Object around Point by Angle” icon, and select the kite and PofR respectively,
- When the dialog box for rotation angle is appeared, type 120 degree and select “clockwise,”
- Repeat the previous step for the angle 240 degree,
- Save your work by naming it as “transformation 1.”

Figure 5. An equilateral triangle obtained by rotating a kite
To have the triangle created dynamically (Figure 4)

- In order to reverse the steps after rotation, simply delete the point PofR. Since all rotations were around the PofR, they will disappear,
- In order to create a slider, click on “slider” icon, and click on the place you want to have your slider located,
- Select the “angle” and then type 120 degree for the “increment” by leaving others as they are,
- Create a point for rotation point, say D
- Activate the “Rotate Object around Point by Angle” icon, and select the kite and the point D, respectively,
- When the dialog box for rotation angle is appeared, type β or β*180/π degree and select “clockwise,
- Right-click on the rotating object and select “trace on,”
- Save your work by naming it as “transformation 2.”

Figure 6. An equilateral triangle obtained by rotating the kite dynamically.
To generalize the construction (Figure 5)

- Create the points A and B,
- Create a line passing through these points,
- Create a slider, by activating “slider” icon and clicking anywhere on the geometry window,
- When the dialog box is opened, select the “integer” and make the minimum and maximum values 3 and 10, respectively (no need to change the increment),
- In order to generalize the angle at the point B, first activate the “Angle with Given Size” icon and then click on the points A and B, respectively. Do not forget to select the “clockwise” after typing 180/n degree in the dialog box,
- Draw a line passing through the points A’ and B,
- Draw a line perpendicular to the line AB at the point A,
- Active the “Intersect Two Objects” icon and select the perpendicular line and the line making 36 degree with the horizontal line (when the slider n shows 5).

Figure 7. Construction of a generalized triangle.
To reflect the triangle over its hypotenuse (Figure 6)

- Make the lines AC and AB, the point A', and the angle α invisible by right-clicking on each and select “Show Object,”
- Select the “Polygon” icon and then click on the points A, B, C and again A to create ABC triangle,
- In order to reflect the triangle ABC about the line BC, activate the “Reflect Object About Line” icon and then click on the triangle ABC and the line BC, respectively.

![Figure 8. Reflection of the generalized triangle.](image)
To have the triangle created dynamically (Figure 7)

- In order to create a slider, click on “slider” icon, and click on the place you want to have your slider located,
- Select the “angle” and then type (360 / n)° for the “increment” by leaving others as they are,
- Create a point for rotation point, say D,
- Activate the “Rotate Object around Point by Angle” icon, and select the kite and the point D, respectively,
- When the dialog box for rotation angle is appeared, type β or β*180/π degree and select “clockwise,”
- Right-click on the rotating object and select “trace on,”
- Save your work by naming it as “transformation 3.”

Figure 9. A polygon dynamically constructed from a kite through rotation.
To have the triangle created dynamically (Figure 8)

- In order to reverse the steps after rotation, simply delete the slider $\beta$. Since all rotations were defined by the slider $\beta$, they will disappear,
- Click on the input line to type commands,
- Start typing “Sequence”, and select “Sequence[ <Expression>, <Variable>, <Start Value>, <End Value> ]” when the options are available,
- For the “<Expression>” part, type “Rotate” and select “Rotate[ <Object>, <Angle>, <Point> ]” when the options are available,
- Replace the expression of “Sequence[Rotate[ <Object>, <Angle>, <Point> ], <Variable>, <Start Value>, <End Value> ]” with “Sequence[Rotate[ poly1, (2\pi i / n), D ], i, 1, n]”
- Repeat the previous steps for “poly2” by replacing the expression with “Sequence[Rotate[ poly2, (2\pi i / n), D ], i, 1, n]”
- Save your work by naming it as “transformation 4.”

Figure 10. A polygon statically constructed from a kite through rotation.
Appendix B: Exploring Geometry in Islamic Art

Examples from the polygons used in Islamic Art

Figure 11. A decoration from Aga Khan Museum, Toronto, ON.

Figure 12. Closer look at the octagon, circled in blue.
Figure 13. A plate from Aga Khan Museum, Toronto, ON.
Figure 14. A close look at the polygons in various number of corners.