LESSON 16: THE CHI-SQUARE DISTRIBUTION

Outline

- Sampling distribution of the variance
- The Chi-Square distribution

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SAMPLING DISTRIBUTION OF THE VARIANCE

- Which supplier is better?
  - One who usually delivers items in a short time, but does not have consistency and often take very long time eventually causing a huge financial loss to the company.
  - Or the one who always delivers items after a long but reasonable period of time.
  - The first supplier's lead time has a smaller mean, but larger variance (or standard deviation).
  - The second supplier's lead time has a larger mean, but smaller variance (or standard deviation).
  - The final decision may be dictated by any of mean or variance.

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SAMPLING DISTRIBUTION OF THE VARIANCE

- We have seen before that sample parameter $\overline{X}$ is used to draw inference about population parameter m. Sample parameter P is used to draw inference about population parameter p.
- Similarly, sample parameter s^2 is used to draw inference about the population parameter s^2.

THE CHI-SQUARE DISTRIBUTION

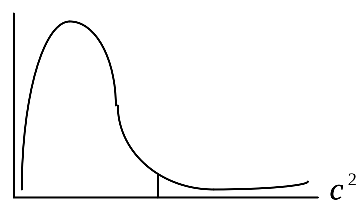
- Assumptions:
  - The chi-square distribution assumes that the sample observations are drawn form a normally distributed population.
  - The chi-square distribution often serves as a satisfactory assumption to the true sampling distribution even when the population is not normal.

THE CHI-SQUARE DISTRIBUTION

- The chi-square statistic

$$c^2 = \frac{(n-1)s^2}{s^2}$$

- The chi-square densities are positively skewed and the shape depends on degrees of freedom, d.f. = n-1. For large (at least 30) d.f. the shape resembles the bell shape as in the case of normal distribution.



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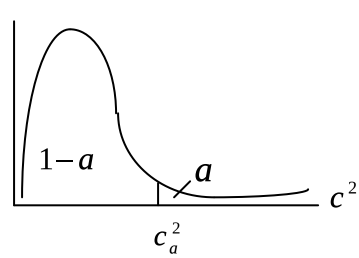
THE CHI-SQUARE DISTRIBUTION

- For each upper-tail probability a, there is a critical value c_a^2 such that

$$a = \Pr\left[c^2 > c_a^2\right]$$

- A large chi-square statistic is usually undesirable. It is inferred at significance level a that the population variance is not less than the assumed value, if $c^2 > c_a^2$



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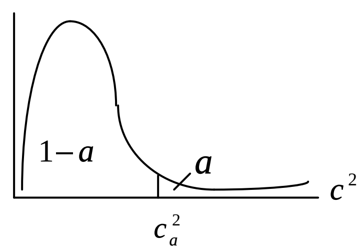
THE CHI-SQUARE DISTRIBUTION

- The $100(1-a)$ percentile for the sample standard deviation is obtained from

$$s = \sqrt{\frac{s^2 c_a^2}{n-1}}$$



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THE CHI-SQUARE DISTRIBUTION

- Table H, Appendix A, pp. 542-543 gives the critical chi-square value c_a^2 for given d.f. and upper tail area a . Notice that the table does not contain values for the large samples. Use normal approximation with

$$E(c^2) = n-1, \qquad \text{SD}(c^2) = \sqrt{2(n-1)}$$

  to Chi-square distribution for large sample (30 or more).
- The relevant Excel commands are CHIDIST and CHIINV
- CHIDIST (critical value, d.f.) returns the upper tail area
- CHIINV (upper tail area, d.f.) returns the critical chi-square value. Thus, CHIINV does the same job as Table H.

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THE CHI-SQUARE DISTRIBUTION

Example 1: Determine the upper-tail critical values for the chi-square statistic in the following cases:

a. $a = 0.05$, d.f. $= 15$

b. $a = 0.20$, d.f. $= 10$

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THE CHI-SQUARE DISTRIBUTION

Example 2: The waiting time of time-sharing jobs for access to a central processing unit is believed to be normally distributed with unknown mean and standard deviation. Find the 70th percentile for the sample standard deviation s when a sample of size $n = 20$ is taken and assuming that the true value of the population standard deviation is $s = 1.5$ minutes.

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THE CHI-SQUARE DISTRIBUTION

Example 3: The mean sitting height of adult males may be assumed to be normally distributed, with mean 35” and standard deviation 1.2”. For a sample size of $n = 100$ men, determine the probabilities for a possible level of the sample standard deviation $s \leq 1.1''$

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READING AND EXERCISES

Lesson 16

Reading:

Section 9-6, pp. 286-289

Exercises:

9-35, 9-36, 9-37, 9-39

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