Abstract

Papers studying the liquidity of a market tend to focus on decisions involving the trade-off between the selling price and the time-till-sale for a given set of market conditions. This paper characterizes the effect of a change in market conditions on a single seller, where seller benefits from either a “value-increasing” or a “liquidity-increasing” change in market conditions. Shamelessly stealing from demand theory, I show how the effect of either type of change on price and on the probability-of-sale can be decomposed into a “value effect” and a “substitution effect.” Two adding-up conditions restrict the set of possible predictions.

The discussion focuses on the market for real estate assets where stochastic rationing is most evident, especially in the discussion of empirical implications. Even so, the same ideas can be applied to markets where other selling mechanisms dominate and, for this reason, I offer a general theory of how individual behavior adjusts to changing conditions.

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Value and liquidity effects

Consider the problem facing someone who has decided to sell their house. The selling price depends on the type of house, on market conditions and on luck. A seller can affect the outcome by carefully choosing the list price that is seen by potential house buyers but that choice also affects the probability of sale and the expected time-till-sale. Researchers are beginning to form a consensus on the best way to estimate the determinants of this second process, but there has been little analysis of the trade-off per se and no analysis of the more fundamental question of how individuals or markets react to a change in market conditions.

The trade off between the selling price and the probability of sale is presumed to exist because the transaction costs associated with buying and selling real estate make it an illiquid asset. This fact would make a real estate market a good choice to study the effects of transaction costs except for two puzzles. First, there is a popular misconception about the degree to which real estate is illiquid. If liquidity is measured by the time taken to raise cash to pay for a random liquidity shock, then that time should not be measured by the many months it takes to sell a house; Even in the extreme case of a seller who owns no other liquid asset, the time may be no more than the few days needed to qualify for a mortgage. Similarly, some common definitions of liquidity appear to be confused when applied to a real estate market. For example, most houses are sold using a matching process and, with a sequential transaction process, any attempt to measure the liquidity of a real estate market in terms of a trade off between selling price and the probability of sale is flawed; there is no direct trade off. More specifically, it is probably true


2 I am indebted to Uday Rajan for this insight.
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that a lower list price attracts more potential buyers, but the selling price is fixed only after the buyer has shown up and only after that buyer and the seller complete their bargaining.

A second problem with using a real estate market to study the effects of transaction costs is that the costs are difficult to quantify directly. Theoretical links between studies of a real estate market and the concept of liquidity are usually weak. Some, e.g. Glower, Haurin and Hendershott (1998), focussed on the effects by focussing on the actions of sellers with different tastes. Since economists usually place few restrictions on tastes, few general predictions were possible. Further, the transaction costs which make the perfectly competitive model inappropriate led economic theorists to consider many different kinds of trading mechanisms where each has characteristics that are not completely understood. Formally, this paper studies the effects of market conditions but, since market conditions change more often than transaction costs, this paper may provide insights into how the mechanisms created to cope with transaction costs adapt to changing conditions of all sorts.

Insert Figure 1

This paper uses a locus of feasible combinations of the expected sale price and of the probability of sale to identify the effects of changing market conditions on a single seller. Figure 1 demonstrates that a change in market conditions, i.e. a change in the locus facing an individual seller, usually has an ambiguous effect. Most observed changes in the locus combine a “value-altering” change and a “liquidity-altering” change where the one of effects of a value-altering change in market conditions is to change the price. But the effect of a liquidity-altering change in market conditions is more subtle because of the pre-existing trade off between price and probability. This paper focusses on decomposing the full effect into a “value effect” and a “substitution effect”. Some adding-up conditions restrict the range of permissible outcomes and
Value and liquidity effects can be used to test whether observed behavior satisfies necessary conditions for optimal decision-making. The following section offers an example showing how the most common specification of regression equations imply restrictions on the seller’s tastes. I also show how this general method can be applied to different types of trading mechanisms.

Many indicators of market conditions have been proposed in informal and formal studies of real estate markets, including

- the unemployment rate or total employment,
- the level of mortgage interest rates,
- the direction or volatility of interest rates,
- the ratio of listings to sales, sometimes known as “inventory”,
- the ratio of new listings to sales, to distinguish “buyer’s market” from “seller’s market”,
- the rate of increase in prices,
- the number of sales,

and, perhaps because of a lack of testable hypotheses and diverse data sources, trend variables. Some indicators suggest that a change primarily alters value or buyers’ willingness to pay (i.e. unemployment rate, interest rate, rate of increase in prices) while other indicators point to a change that primarily alters the flow of buyers and a seller’s probability of sale (i.e. volatility of interest rates, inventory, sales). It is also possible to classify indicators as internal or external where, for example, the number of sales is an internal indicator because it can be expected to change as a market adjusts. External indicators of market conditions in a real estate market, such as interest rates or the unemployment rate, do not adjust as a single market adjusts. Lastly, it may be possible to make the aggregated data typically reported by real estate boards more relevant to an individual seller by using segment-specific indicators.
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Whether any of these classification schemes is informative should be decided by empirical analysis. These classifications are not relevant to this paper since I study how a single seller who reacts to the conditions in the market, however they may be determined. The conclusion discusses how the behavior of individuals can be integrated into an equilibrium model.

Finally, a real estate market offers two important advantages over another market commonly used to study transaction costs: a labor market. Though few people use the word “liquidity” to characterize a labor market, the models used to study unemployment behavior (e.g. Pissarides, 2000) and those used to study the real estate selling process (Yavas, 1992) are both based on search and matching models. An important difference is that most of the data sets used to study labor market data typically include information on only the final outcome while the list price recorded by data sets on real estate markets can reveal information about the seller during the transaction process (Knight, 2002). Second, studies of labor markets indicate that nearly all job offers are accepted (Devine and Keifer, 1991). Studies of real estate markets indicate that many of the houses offered for sale do not sell (Anglin, Rutherford and Springer, 2003) and that many negotiations between a buyer and a seller do not reach an agreement (Anglin, 1994). Thus, house sellers seem to explore more margins more actively than workers.

Decomposing the Effect

Notation

As a decision problem involving a large fraction of their net wealth, most house sellers desire to make an optimal decision. A seller’s tastes are represented by $U(p^S, \lambda)$ where $U(.)$ is assumed to be increasing in $(p^S, \lambda)$ and is strictly quasi-concave. For a given type of house and
given market conditions, \( X \), the locus of feasible combinations of price-probability is defined by

\[
(p^s(X, p^l), \lambda(X, p^l))
\]

where \( p^s \) represents the expected selling price, \( \lambda \) represents the probability of sale during a period of time and \( p^l \) represents the list price that can be chosen.\(^3\)

By removing the common parameter, \( p^l \), the locus can be represented as

\[
F(p^s, \lambda, X) = 0.
\]

A linear approximation to \( F(.) \) simplifies the analysis:\(^4\)

\[
F(p^s, \lambda, X) = -\nu + p^s - \tau (1 - \lambda) = 0 \quad \text{or} \quad p^s = \nu + \tau (1 - \lambda)
\]

for some \((\nu, \tau)\), which depend on \( X \). \( \tau > 0 \) ensures that the locus is downward sloping while \( \nu \) shows what a seller could obtain in an instantaneous sale, i.e. \( \lambda = 1 \). A change in market conditions corresponds to a change in \( \nu \), which defines a value-altering change, or to a change in \( \tau \), which defines a liquidity-altering change, or to change in both.

The fact that my analysis focusses on payoff-relevant variables means that the analysis can incorporate features that are specific to a market with few changes. For example, if a seller

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\(^3\) In a continuous time model, with a constant hazard rate, \( \lambda \), and a constant discount rate of \( r \), the expected present value of selling a house satisfies \( U(p^s, \lambda) = p^s \lambda/(\lambda + r) \). This derivation reveals two insights: First, even if sellers behave according to the precepts of expected utility, the relevant objective function can be non-linear. Second, although the expected time-till-sale in a stationary environment would be \( 1/\lambda \), the fact that market conditions can change suggests that the relevant environment is not necessarily stationary. Thus, estimates of the probability of sale at any point in time may be more stable than estimates of the expected time-till-sale.

\(^4\) There may be good reasons to expect the locus to be non-linear and interested readers may find the work of Blomquist (1989) on the implications of comparative statics with linear and non-linear budget constraints helpful. He found that many, but not all, of the properties commonly associated with demand theory continue to hold when using “local prices” and “virtual income”. To the extent that any implication is an empirical question, Hausman (1985) and Moffitt (1990) offer introductions to the kinds of techniques needed.
Value and liquidity effects has more than the one decision variable presumed in the presentation above, such as list price and the effort of a real estate agent (Yavas and Yang, 1995) or list price and bargaining position (Arnold, 1999), then the relevant locus is the upper envelope of a set of one-variable loci in price-probability space. Any change in market conditions affects a seller’s behavior only if it affects the upper envelope.

The locus can be generated from many types of models. Footnote 1 documents a large empirical literature which is based on this relationship. Wolinsky (1986) studied how a market characterized by imperfect information could appear to be monopolistically competitive market when each firm chooses a posted price. McAfee (1993) and Peters (1997) discussed the kinds of mechanisms that can be supported in a market where buyers and seller compete to attract traders. If the seller has no choice, then the price-probability locus is a single point and the effect of any change can be found mechanically.

Since the choice of list price affects the seller’s utility only if it affects \((p^S, \lambda)\), a seller seeks to solve

\[
\max_{(p^S, \lambda)} U(p^S, \lambda) \quad \text{s.t. } -v^+ p^S - \tau (1 - \lambda) = 0
\]  

(4)

with the solution being \((p^S(\tau, \tau), \lambda^*(v, \tau))\). The assumptions stated above ensure that the second order conditions are satisfied and that there exists a unique solution. From this solution, one can derive the optimal list price. The dual to this primal problem is

\[
\min_{(p^S, \lambda)} p^S - \tau (1 - \lambda) \quad \text{s.t. } U(p^S, \lambda) = Z
\]  

(5)

with the solution being \((p^S_c(\tau, Z), \lambda^c(\tau, Z))\). The superscripts in these solutions indicate that the seller is compensated for any change in market conditions to achieve the same level of utility. Define the Probability-Adjusted Price, \(PAP(\tau, Z)\), as

\[
PAP(\tau, Z) = p^S_c(\tau, Z) - \tau (1 - \lambda^c(\tau, Z)).
\]  

(6)
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In the special case of $\lambda = 1$, PAP equals the expected selling price.

Relationships

Optimization creates a relationship between the solutions to the primal and dual problems. Proposition 1 exploits this relationship.

Proposition 1.

i) $PAP(\tau, Z) = v$.

ii) $\lambda^*(\tau, Z) = \lambda^*(PAP(\tau, Z), \tau)$ and $p^{sc}(\tau, Z) = p^{s}(PAP(\tau, Z), \tau)$.

iii) $\frac{\partial \lambda^*}{\partial \tau} = -(\frac{\partial \lambda^*}{\partial v}) (1 - \lambda^*) + \frac{\partial \lambda^*}{\partial \tau}$.

iv) $\frac{\partial p^{sc}}{\partial \tau} = -(\frac{\partial p^s}{\partial v}) (1 - \lambda^*) + \frac{\partial p^s}{\partial \tau}$.

Proof

i) The feasible set is $0 = -v + p^{s} - \tau (1 - \lambda)$. Substituting the solution to the dual problem proves that

$$0 = -v + p^{sc} - \tau (1 - \lambda^*) = -v + PAP.$$  \hspace{1cm} (7)

The claim follows immediately.

ii) This claim is proven by substituting part i) into the solutions to the primal problem and noting that the solutions must be identical since the primal and dual problem are different methods to the same answer (see, for example, Varian, 1984, ch 3).

iii) and iv): The proof uses the identities proven in ii) and the now-standard method of deriving the Slutsky Equation first presented by Cook (1972). Q.E.D.

To complete the analogy to the Slutsky Equation, this Proposition shows that every value-increasing change in market conditions affects the seller’s choice of price and probability, $(\frac{\partial \lambda^*}{\partial v}, \frac{\partial p^s}{\partial v})$. Every liquidity-increasing change in market conditions has both substitution effects $(\frac{\partial \lambda^*}{\partial \tau}, \frac{\partial p^{sc}}{\partial \tau})$ and value effects, $((\frac{\partial \lambda^*}{\partial v}) (1 - \lambda^*), (\frac{\partial p^s}{\partial v}) (1 - \lambda^*))$. If there are only
two dimensions to a seller’s problem then Figure 1 shows that the substitution effect associated with an increase in liquidity causes a seller to decrease the probability of sale and increase the expected selling price.

The most obvious example of a value-increasing change is a change in house type: \(v\) depends on the characteristics of the house. Even if \(d\tau/dX=0\), a seller of a more valuable house can charge a higher price or sell with a higher probability or both, ceteris paribus, because their price-probability locus is higher. An increase in interest rates may represent a different kind of value-altering change in market conditions since the increase affects a buyer’s pay for a mortgage and affects their willingness to invest in assets other than real estate. Similarly, an unemployed buyer would tend to be willing to pay less than an employed buyer. To the extent that all of these variables affect \(v\), evidence on any one effect should reveal the effect of all other value-altering changes.

At the same time, an increase in interest rates may discourage some buyers from looking to buy. Uncertainty about interest rates or an increase in the number of competing sellers would tend decrease the flow of buyers to any one seller and reduce the probability of sale for that one seller. The magnitude of this effect is an empirical question but the ability to combine changes in many variables into a liquidity-altering change simplifies the analysis.

Regardless of tastes, feasibility of the optimal solution implies

\[
-v + p^*(v, \tau) - \tau (1 - \lambda^*(v, \tau)) = 0. \tag{8}
\]

Differentiating equation (8) with respect to \(v\) and \(\tau\) leads to two conditions which are valid for all tastes:

\[
\frac{\partial p^*}{\partial v} + \tau \frac{\partial \lambda^*}{\partial v} = 1 \tag{9}
\]

\[
\frac{\partial p^*}{\partial \tau} + \tau \frac{\partial \lambda^*}{\partial \tau} = 1 - \lambda^*. \tag{10}
\]
And, given the conditions imposed above, \( p^* \) and \( \lambda^* \) are differentiable everywhere except at the boundary: \( \lambda^* = 1. \) Thus,

**Corollary 1.**

i) If \( \lambda^* < 1, \) \( \partial p^*/\partial v = 1 \) if and only if \( \partial \lambda^*/\partial v = 0. \)

ii) \( \partial \lambda^*/\partial \tau = 0 \) if and only if \( \partial p^*/\partial \tau = 1 - \lambda. \)

**Example and Empirical Applications**

The papers cited in the bibliography note that many analysts who study real estate markets now prefer to use a log-linear model to explain the selling price and a proportional hazards model to explain the time-till-sale. That is,

\[
E(\log(p^*)) = X\beta \\
\lambda = \exp(X\gamma).
\]

where \( X \) represent the explanatory regressors, and \( \beta \) and \( \gamma \) represent parameters to be estimated.

If valid, this simple two-equation system produces a unique solution for a seller’s implied indifference curves. It also suggests a way to make the estimated coefficients more precise.

To focus on \( (v, \tau) \), and since \( (v, \tau) \) changes only if \( X \) changes, it helps to restate the right hand side of these equations:

\[
E(\log(p^*)) = A v + B \tau \\
\log(\lambda) = C v + D \tau.
\]

for some parameters, \( A, B, C \) and \( D. \) It also helps to express \( (v, \tau) \) as functions of \( (p^*, \lambda): \)

\[
v^0 = \pi_1 E(\log(p^*)) + \pi_2 \log(\lambda) \\
\tau^0 = \pi_3 E(\log(p^*)) + \pi_4 \log(\lambda)
\]

where \( (v^0, \tau^0) \) represents the values which produce \( (p^*, \lambda) \) as the optimum and
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$$\pi_3 = -\frac{C}{(A - D - B C)}$$ (18)

$$\pi_4 = \frac{A}{(A - D - B C)}.$$ (19)

Optimization implies that a seller’s indifference curve in \((p^S, \lambda)\) space is tangent to the price-probability locus:

$$\frac{dp^S}{d\lambda} \bigg|_{\lambda = u} = -\tau^0(p^S, \lambda)$$ (20)

Thus, solving for a seller’s indifference curve involves solving a differential equation (Varian, 1984, Sec. 3.10). If \(\pi_3 = C = 0\) then combining equations (17) and (20) reveals that the differential equation is

$$dp^S = -\pi_4 \log(\lambda) \, d\lambda$$ (21)

and the seller’s indifference curve must satisfy

$$p^S = L - \pi_4 (\lambda \log(\lambda) - \lambda)$$ (22)

for some constant \(L\) to be determined by an initial condition. If \(\pi_4 = A = 0\) then the relevant differential equation is

$$dp^S = -\pi_3 \log(p^S) \, d\lambda$$ (23)

and the associated indifference curve satisfies

$$-\pi_3 \lambda = L + \log(\log(p^S)) + \log(p^S) + (\log(p^S))^2/(2 \cdot 2!)+ \ldots$$ (24)

Different tastes produce different expansion paths in \((p^S, \lambda)\) space and, if variation in \(A, B, C, D\) spans an incomplete range of tastes then equations (12) and (13) may impose unintended restrictions. It may be better to start a flexible functional form, such as an Almost Ideal Demand System or a Generalized Leontief, which produces regression equations for \((p^S, \lambda)\) that are easy

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5 I ignore the detail that \(E(\log(p^S))\) usually differs from \(\log(E(p^S))\). When this difference is caused by variables omitted from a data set but observed by a buyer and a seller, this difference has no effect.
One common use of equation (13) is to study whether different list prices affect the probability of sale. For example, Anglin, Rutherford and Springer (2003) included a measure of the Degree of Over-Pricing, DOP, on the right hand side of the regression equation:

$$\log(\lambda) = C + D + E \text{DOP.}$$

(25)

for some constant E. Though rarely estimated, this equation is paired with another showing the effect on the selling price:

$$E(\log(p^S)) = A + B + F \text{DOP}$$

(26)

for some constant F. If the model is consistent then a seller of type DOP should have an indifference curve that satisfies

$$\frac{dp^S}{d\lambda}|_u = -\pi_3 (E(\log(p^S)) + F \text{DOP}) - \pi_4 (\log(\lambda) - E \text{DOP}).$$

(27)

We can rank sellers’ tastes in the obvious way if and only if an increase in DOP increases the expected selling price and decreases $dp^S/d\lambda|_u$: i.e. $F > 0$ and $\pi_3 F + \pi_4 E = (A F - C E)/(A D - B C) < 0$.

This example focuses on a trade off between the expected selling price and the probability of sale but a seller may also be concerned about price risk or time risk. Solving for a seller’s utility function to this broader problem are possible but they involve solving some partial differential equations (Mas-Colell et al, 1995, Sec. 3.H). More useful predictions about behavior exist since a Slutsky-like Equation remains valid regardless of the number of arguments in the utility function. For example, some types of sellers may value the higher moments of the distribution of selling price because of risk aversion or loss aversion (Genesove and Mayer, 2001); suppose that this risk depends on $p^L$. This addition leads to market conditions being described by a price-probability-risk locus and a seller who wishes to choose the preferred point
on it. Whereas a risk-neutral seller is indifferent between all extensions of a given price-probability locus, a risk averse seller’s optimal solution would depend on the restrictions imposed by a particular extension of the price-probability locus. Thus, application of the Le Chatelier Principle shows that sellers who are concerned about higher moments of the selling price distribution should be less sensitive to changes in market conditions. More formally,

**Proposition 2**

\[
\begin{align*}
&i) \frac{\partial \lambda^*}{\partial v} \geq \frac{\partial \lambda^*}{\partial v}|_R \quad \text{and} \quad \frac{\partial \lambda^*}{\partial \tau} \geq \frac{\partial \lambda^*}{\partial \tau}|_R \\
&ii) \frac{\partial p^S}{\partial v} \geq \frac{\partial p^S}{\partial v}|_R \quad \text{and} \quad \frac{\partial p^S}{\partial \tau} \geq \frac{\partial p^S}{\partial \tau}|_R
\end{align*}
\]

where \( |_R \) denotes the restriction that the solution must belong to the price-probability-risk locus.

**Other Markets and Other Mechanisms**

I used the example of selling a house to motivate the value and substitution effects of a change in market conditions on a single seller. Different markets use different mechanisms and many papers have studied these mechanisms under a variety of given market conditions. This section notes how ideas developed in the context of the search and bargaining mechanism used to sell real estate might apply equally well to other types of mechanisms used to sell other types of goods. I show that each mechanism is the same in the sense that a seller controls some feature of the mechanism and that choice implies some expectation for the probability of sale and the selling price. Thus, a change in market conditions changes the choice in easy-to-understand ways and, indirectly, this fact demonstrates that value and substitution effects offer a unified approach to studying many types of markets.

**Auctions**

Auctions have been advocated as ideal mechanisms to sell a good whose value is
uncertain and existing theory has sought to understand the properties of auctions. To illustrate the price-probability locus relevant for an auction, consider a second-price auction. In this type of auction, a seller chooses a reserve price, B, which is assumed to be publicly known and assumed to be binding, and a time to hold the auction, T. The item is sold to the highest bidder at a selling price determined by the greater of the reserve bid and the second highest bid. It is well-known that a buyer’s dominant strategy is to bid their willingness-to-pay.

More formally, suppose that potential buyers arrive independently at a constant rate of \( \lambda \) per unit of time and that their willingness-to-pay is represented by the cumulative distribution function \( G(p) \). By selling at the second highest price, the cumulative distribution of selling prices is

\[
\frac{n(G(\max(p^S, B)))^{n-1} - (n-1) (G(\max(p^S, B)))^n}{n}
\]

where \( n \) is the number of bidders at the time of auction (Phillips, 1988). \( n \) is distributed according to a Poisson distribution with mean equal to \( \lambda T \). The ex ante probability that at least one buyer is willing to bid at least B at time T is

\[
1 - E[(G(B))^n; n \sim \text{Poisson}(\lambda T)].
\]

By considering different values of B and T, one can trace out a locus showing the trade off between the expected selling price and the ex ante probability. Thus, the conclusions of Proposition 1 apply. If no buyer in the auction is willing to pay more than B, then the seller must consider the value of not selling. The next section discusses how such dynamic aspects might be added to the model.

**Posted Price Mechanism**

Selling goods by posting a price is special because the posted price is both the list price and the selling price. The literature on menu costs, e.g. Barro (1972) or Benabou and Konieczny
A price-probability locus exists for a posted price mechanism and is usually expressed as the probability of finding a single buyer willing to pay the posted price. Knowing $\hat{\lambda}(p^s)$, ($\nu$, $\tau$) can be fitted to equation (3) with the following defining equations:

$$\tau = 1/(d\hat{\lambda}/dp^s)$$  \hspace{1cm} (31)

$$p^s = \nu + (1- \hat{\lambda})/(d\hat{\lambda}/dp^s).$$  \hspace{1cm} (32)

Much of the interest in the posted price format arises from the fact that, over a long period of time, the probability of finding a single buyer is less important than the flow of sales during a period $\Delta$. If buyers act independently then the average flow of sales is $Q(p^s) = \Delta/\hat{\lambda}(p^s)$ and the most obvious measure of a seller’s utility is the flow of profit: $(p^s - MC) (\Delta/\hat{\lambda})$ where $MC$ represents the marginal cost.

Financial Markets

Financial markets are a special example of a market where auctions are common. The auction operates continuously, the price evolves over time and these features combine to create a price-probability locus. They are also special because the intercept of the price-probability locus is especially easy to observe: a bid represents the price that a buyer is willing to pay now to trade immediately.

To illustrate one version of the price-probability locus, suppose that the market price evolves according to a simple binomial process: the market price starts at $p_0$ and changes each period by either moving up by a factor $u$ or down by a factor $d$, where $ud = 1$. Suppose also that the per-period probability of an increase is independent across periods and equals $\theta$. In this environment, a seller chooses a trading rule to sell a financial asset which can be as simple as selling at the market price on a pre-specified date, $t$. Or, the rule can fix a selling price, $S$, and
wait until the market price rises to that level. The expected selling price under the first rule equals the expected market price after $t$ periods: the expected price at time $t$ is $p_0 d^t E((u/d)^x)$ where $x$ is randomly distributed according to the Binomial($t, \theta$) distribution. Deriving the probability of sale under the second rule is more complicated since it must recognize the possibility of sale at some time before period $t$ but, with a computer, the calculation is feasible. These rules reveal the price-probability locus because different trading rules produce different outcomes and the price-probability locus represents the undominated outcomes. The rule chosen by a particular seller depends on their tastes and the prevailing market conditions. Proposition 1 show the effects of a change in market conditions on the observable variables: the expected selling price and the probability of sale. A change in market conditions is revealed by a change in $u$, $d$ or $\theta$ and a change in any of these parameters can be expected to change the seller’s preferred trading rule.

Weill (2003) used a different foundation to produce a linear locus in a market equilibrium for illiquid financial assets. He used it to calculate the variation in asset returns that can be attributed to differences in liquidity. Even so, recent research on the liquidity of financial markets suggests that there may be more than one dimension to the concept of liquidity. For example, D’Souza, Gaa and Yang (2003) discussed four dimensions of liquidity: tightness, immediacy, depth and resiliency. The price-probability locus focusses on the trade off between the ability to sell immediately and the required change in price (i.e. tightness). The “depth” of a market reveals the aggregate number of orders at a given time and “resiliency” reveals whether any disturbance to the normal flow of orders has long-lasting consequences. Associated with each dimension is one or more measures that have been shown to be significant, such as the bid-ask spread, trading volume and price-impact coefficients. These measures represent the
aggregated effects of different types of traders, such as “market makers”, “noise traders”, “informed traders” who place differing weights on the different dimensions, whose actions combine to make a market more or less liquid in ways that each single seller must adapt to when making a choice.

Financial markets differ from real estate markets in one final way: a seller may sell some part of a divisible asset and remain in the market. This fact means that traders may use sequential strategies to exploit, or to prevent being exploited by, asymmetric information. A few papers have studied markets where a trader is “informationally small” in addition to being one of a large number of similar traders (McLean and Postlewaite, 2002). In such markets, focussing on a price-probability locus may be sufficient to explain the behavior of an individual trader.

Dynamic Models

The previous discussion focussed on a familiar static model where any change is anticipated and permanent. Many people seek to understand the effects of changing market conditions because the conditions are not permanent and sometimes not anticipated. For example, it usually takes months to sell a house and interest rates can change during this time. During price negotiations, the decision to sell or to wait for better conditions may vary with conditions that evolve over time (Krainer, 1999; Novy-Marx, 2003). In financial markets, traders try to profit from the constant flow of new information about market conditions. These facts make a static model unrealistic. Fortunately, the fact that a seller values only the selling price and the probabilities of sale at a sequence of points in time allows me to analyse this problem without invoking the full complexity of a dynamic Slutsky Equation (LaFrance and Barney, 1991; Caputo, 1990). Thus, with a few modifications, the results derived above remain
Without loss of generality, suppose that time flows discretely, \( t = 1, 2, \ldots \), and that anticipated market conditions vary with the date, \((v_t, \tau_t)\). Since a seller maximizes utility at each point in time, subject to the sequence of feasible price-probability loci \(\{(v_t, \tau_t); t = 1, 2, \ldots\}\), a Bellman Equation reveals the relevant objective function at any point in time:

\[
V_t(p^s_t, \lambda_t; v_{t+1}, \tau_{t+1}) = \lambda_t U(p^s_t) + (1 - \lambda_t) \max \{V_{t+1}(p^s_{t+1}, \lambda_{t+1}; v_{t+2}, \tau_{t+2}); (p^s_{t+1}, \lambda_{t+1}) \text{ are feasible}\} \tag{33}
\]

With this expression and market conditions in period \( t \) summarized by

\[
p^s_t = v_t + \tau_t (1 - \lambda_t), \tag{34}
\]

Proposition 1 can be applied directly to find the effect of a change in period 1’s market conditions on behavior during period 1. In the same way, Proposition 1 reveals the effect of an anticipated change in market conditions in period \( t+1 \) on behavior during period \( t+1 \) if the seller survives until period \( t+1 \).

A change in market conditions during any period later than \( t \) affects \( V_{t+1} \) and the effect on behavior during earlier periods is comparable to a change in tastes in a static model. Proposition 3 provides a more precise analysis of this change.

**Proposition 3**

\[
\frac{d\lambda_t}{dV_{t+1}} = 1/\left( -\tau_t U' + \tau_t^2 U'' \right) < 0
\]

\[
\frac{dp^s_t}{dV_{t+1}} = -\tau/\left( -\tau_t U' + \tau_t^2 U'' \right) > 0
\]

\[\text{6 If changes in market are not anticipated then they would have no effect on the seller’s behavior: for a fixed list price, the effect of a change in the locus can be inferred mechanically from equations (1) and (2).}\]
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if the seller is not risk loving.

Proof:

By optimizing sequentially, the effect of changes in market conditions after period t can be accurately summarized by $V_{t+1}$. Thus, substitution of the price-probability locus into equation (33) produces a simple objective function

$$
\lambda_i U(v_i + \tau_i(1 - \lambda_i)) + (1 - \lambda_i) V_{t+1}
$$

and a simple first order condition:

$$
0 = U(v_i + \tau_i(1 - \lambda_i)) - V_{t+1} - \tau_i U'(v_i + \tau_i(1 - \lambda_i)).
$$

Totally differentiating equation (35) with respect to $V_{t+1}$ and simplifying implies

$$
0 = -dV_{t+1} + d\lambda_i \left\{ -\tau_i U'(v_i + \tau_i(1 - \lambda_i)) + \tau_i^2 U''(v_i + \tau_i(1 - \lambda_i)) \right\}
$$

Since $\tau_i > 0$ for all t, the claim follows directly. For given t, and $(v_i, \tau_i)$, equation (33) shows that if $\lambda_i$ rises then $p_{S_i}^*$ falls. Q.E.D.

If market conditions evolve according to a Markov process, where $D((v_{t+1}, \tau_{t+1}); (v_t, \tau_t))$ denotes the distribution of $(v_{t+1}, \tau_{t+1})$ conditional on $(v_t, \tau_t)$, then modifying the right hand side of equation (33) to recognize the conditional expectation in each period would produce results comparable to Propositions 1 and 3. Quantifying these effects requires simultaneously estimating the value effect, substitution effect and $D(.)$.

An Aside on Asset Price Bubbles

These ideas offer some insights into the existence of price bubbles in assets, such as real estate. One of the reasons why economists are fascinated by the phenomenon of a bubble may be our preference for theories of price-taking behavior and that the price seems to be given “wrongly”. As usually described, a bubble occurs when a price increase is supported by an expectation of future price increases. Eventually, the price level becomes sufficiently high that it
becomes unsustainable, the bubble “bursts” and prices to return to a more normal level. From the perspective of an individual seller, this story focuses on the value-altering dimension of market conditions. This story ignores the fact that illiquidity gives a seller some control over the price and that the selling price of a house may change because of a liquidity-altering change in market conditions.

Media descriptions of bubbles talk often about panic buying during the later stages of a bubble or about houses being sold within a few days or hours of being listed. While recognizing the excited writing style and selective use of data associated with daily media reports, these descriptions should be considered as a relevant part of the process with predictable implications. When a seller expects a much higher value of $\tau$ than under normal conditions, Proposition 1 shows the associated effects and that the magnitude of the effect falls as $\lambda^*$ rises. Ceteris paribus, markets with a lower normal probability of sale are likely to see a stronger link between changes in $\tau$ and changes in value.

Understanding that a change in $\tau$ also changes the realized selling price, with a certain probability, suggests a difference between the early stages of a bubble and the later stages. When the selling price rises because of the substitution induced by an increase in $\tau$, and especially if the selling price is not rising but there is evidence that the seller’s anticipated value of $\tau$ is increasing, then $\nu$ may be falling. I conjecture that a permanent increase in $\tau$ is not possible because it would require a permanent change in the ratio of buyers to sellers. The price bubble bursts when the flow of buyers falls, causing sellers’ anticipated values of $(\nu, \tau)$ to become more realistic and more compatible with the actions of other players in the market. The concluding section offers some thoughts on characterizing a steady state equilibrium in the market.
Concluding Comments

From the perspective of a single seller, changes in market conditions can be reduced to some combination of two types: value-altering and liquidity-altering. This paper shows that, under a variety of different selling mechanisms, any effects on payoff-relevant dimensions of a seller’s behavior can be summarized by a combination of a value effect and a substitution effect. These effects impose cross-equation restrictions that can be exploited to improve the efficiency of a chosen econometric procedure or to test whether sellers are making an optimal decision.

The proofs of these results use ideas commonly associated with demand theory, especially the Slutsky Equation. This Equation is a powerful tool because it can be the basis for the complete set of testable predictions (e.g. Mas-Colell, Green and Whinston, 1995). By focussing on payoff-relevant aspects of behavior, I note how a study of value and substitution effects offers insights into the market process in many types of markets.

The magnitude of each effect may vary according to the mechanism or the market but I offer a simple example where a seller’s preferences could be derived from some common specifications of regression equations. With the few exceptions noted above, most reports of these regression equations exclude the measures of market conditions noted in the introduction. They could, and should, be added to the regression equations. If a linear price-probability locus is found to be unsuitable then Hausman (1985), Moffitt (1990) and others have developed the econometric technology needed to do better. In practice, implementing these equations might reveal a conceptual ambiguity: does the estimated price function reveal an equilibrium relationship between price and quality as derived by Rosen (1974) or does it also reveal one dimension of a seller’s trade off between price and probability-of-sale. Resolving this ambiguity depends on having data on a relevant cross-section of sellers and on appropriately recognizing
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the interaction between the measured market conditions and the actions of individual sellers.

I considered both a static model and a dynamic model. Two interesting non-Markovian
dynamic processes may also be worth considering. The more obvious process represents a
correlation between market conditions during one period and conditions during later periods;
Proposition 3 suggests that the degree of correlation would affect the effects observed for any
given change in exogenous conditions. A slightly less obvious process represents the
phenomenon of stigma. Taylor (1999) found an equilibrium to an asymmetric information game
where the price-probability locus in each period varies over time because an item which has been
offered for sale for a longer time is believed to be of lower quality.

Over long periods of time, the dynamics and the prevailing market conditions will be
constrained by equilibrium conditions. This paper focussed on the question of how a single
seller responds to a change in market conditions because the general idea of how a market
adjusts to changing external conditions is well-known: A market adjusts by

changing the price that each seller offers (and each buyer bids) and by
changing the selection of participants in a market.

Though these changes must be compatible in an equilibrium, they need not be compatible at
every instant in time. A stochastic rationing process, such as that which exists in a real estate
market and is implicit in the discussion above, implies that the first process is always relevant.
Demand theory offers a concept which could be used to study the second process: the indirect
utility function. If \( U(\cdot; \sigma) \) is the utility function of a type \( \sigma \) seller then

\[
W(v, \tau, \sigma) = U(p^{s*}(v, \tau, \sigma), \lambda^*(v, \tau, \sigma); \sigma) \tag{38}
\]

shows the value of that type of seller actively trying to sell under market conditions described by
\((v, \tau)\). Using this function and a modified Roy’s Identity, it may be possible to discover a
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general model of selection which would establish whether the price and probability choices of active sellers complement or offset shocks to the market.
Bibliography


Glower, M., D. Haurin, P. Hendershott, 1998. “Selling time and selling price: The influence of


Figure 1: The Effect of a Change in Market Conditions on the Price-Probability Locus