7. CYCLONE (C&A Chapter 4)

7.1 BASIC PRINCIPLE

- Air containing particle is forced to swirl around
- Centrifugal force pushes particles to outside of vessel where they impact with wall, accumulate, and fall out

7.1.1 REVERSE FLOW (C&A Fig 4.1)

- Most common and efficient
- Air reverses direction but particles can’t

7.1.2 NON-REVERSING

- Lower pressure drop than reverse flow
- Not as effective at removing small particles as reverse flow

7.1.3 ADVANTAGE OF CYCLONE SEPARATORS

- Low capital cost
- Non-proprietary (standard designs are published, e.g. C&A Table 4.1)
- Can operate at high temperatures, pressures, high dust concentrations
- Dust collected is clean and not contaminated by filter fibers → reusable (e.g. pharmaceuticals)

7.1.4 DISADVANTAGES OF CYCLONE SEPARATORS

- Can’t be used with “sticky” dust (tacky and staticy)
- Not recommended when condensation will occur in cyclone (solution: heat & insulate)
• Poor removal of particles < 5 µm
• High pressure drop → high energy use

7.2 COLLECTION EFFICIENCY

1. Number of revolutions, \( N_e \)

\[
N_e = \frac{1}{H} \left[ L_b + \frac{L_c}{2} \right]
\]

\( H \) = height of inlet duct (m)
\( L_b \) = length of cyclone body (m)
\( L_c \) = length of cone (m)

2. Length of time air is in cylinder, \( \theta \)

\[
\theta = \frac{\text{distance traveled}}{\text{inlet velocity}} = \frac{\pi D Ne}{v_i}
\]

3. Minimum radial velocity of particle to be collected

\[
v_{\text{min}} = \frac{W}{\theta}
\]

Drag Force = \[
\frac{1}{2} \left( \pi r_p^2 \right) \frac{\rho f V_{r,p}^2}{g_c} C_d'
\]

Settling chamber H

\( v_{y,p} = v_i = \frac{H}{\theta} \) for removal

Cyclone
Centrifugal Force = \( \frac{m_p v_{t,p}^2}{g_c R} \)

Buoyant Force = \( \frac{m_f v_{t,p}^2}{g_c R} \)

Assume
- \( C = 1.00 \)
- Radical flow around particle is laminar
- Particle quickly reaches terminal velocity in the radial direction
- Particle is spherical

\[
F_D = \frac{1}{2} \left( \pi r_p^2 \right) \frac{\rho_f v_{r,p}^2}{g_c} \frac{24}{Re} \left( \frac{1}{2} \pi r_p^2 \frac{\rho_f v_{r,p}^2}{g_c} \right) \frac{24\mu}{\rho_r v_{r,p} 2r_p} = \frac{6\pi r_p v_{r,p} \mu}{g_c}
\]

Assume
- Tangential velocity of particle = tangential velocity of air
  - = inlet velocity

\[
F_C - F_B = \frac{4}{3} \pi r_p^3 (\rho_p - \rho_f) v_i^2
\]

Since particle is at constant (terminal) velocity, \( F_D = F_C - F_B \)

\[
\frac{6\pi r_p v_{r,p} \mu}{g_c} = \frac{4\pi r_p^3 (\rho_p - \rho_f) v_i^2}{3g_c R}
\]

\[
v_{r,p} = v_t = \frac{2r_p^2 (\rho_p - \rho_f) v_i^2}{9\mu R} = \frac{d_p^2 (\rho_p - \rho_f) v_i^2}{18\mu R}
\]
\[ v_{\text{min}} = \frac{W}{\pi D N e} = v_t \]

\[ d_p^2 = \frac{Wv_i}{\pi D N e (\rho_p - \rho_f) v_i^2} \frac{9D\mu}{\pi} \]

\[ \therefore d_p = \sqrt{\frac{9W\mu}{\pi Ne(\rho_p - \rho_f)v_i}} \]

4. But all particles > \( d_{p_{\text{min}}} \) will not be removed
   
   - approach of Lapple (1951) is to consider particles of “critical” size that will be removed 50%

   \[ v_c = \frac{W/2}{\theta} \]

   \[ d_{p_c} = \text{diameter of particle removed with 50\% efficiency (m)} \]

   \[ = \left[ \frac{9\mu W}{2\pi Ne v_i \rho_p} \right]^{1/2} \quad \text{(C&A Eqn.4.5)} \]

5. Other size particles

   \[ \eta_j = \frac{1}{1 + \left( \frac{d_{p_c}}{d_{p_j}} \right)^2} \quad \text{(also in Figure 4.5)} \]

   \[ \eta_j = \text{efficiency of removal of particle j} \]

   \[ d_{p_j} = \text{diameter of particle j} \]
7.3 SALTATION VELOCITY

Collection efficiency sometimes observed to decrease with increase in inlet velocity. Why?

Definition:

a) Minimum fluid velocity to prevent settling-out of the solids
b) Velocity required to pick up particles and transport them without settling

- The Kalen & Lenz (1974) relationship was applied by Koch & Licht (1977):

$$v_s = 2.055\omega\left[\frac{(W/D)^{0.4}}{(1 - W/D)^{0.333}}\right]^{0.067}D^{0.67}v_i^{0.67}$$

where

$$\omega = \sqrt[3]{\frac{4g\mu(\rho_p - \rho_f)}{3\rho_f^2}}$$

W = inlet width (ft)
D = cyclone diameter (ft)
v_i = inlet velocity (ft/s)
\(\mu = \) viscosity of fluid (lbm/ft.s)
g = 32.2 ft/s^2

Note: Those are empirical equations.

.: use specified units (ft, lbm, s) only

- Max \(\eta\) occurs around \(v_i/v_s = 1.25\)
- Re-entrainment occurs at \(v_i/v_s = 1.36\) (check for this in design)
7.4 PRESSURE DROP

- Recall Bernoulli’s principle:

\[
\begin{align*}
\frac{P_1}{\text{static press.}} + \frac{1}{2} \rho \left( \frac{v_1}{\text{velocity press.}} \right)^2 + Z_1 \rho g &= \frac{P_2}{\text{barometric press.}} + \frac{1}{2} \rho \left( \frac{v_2}{\text{barometric press.}} \right)^2 + Z_2 \rho g
\end{align*}
\]

Express in terms of heights or “heads”

\[
\begin{align*}
\frac{P_1}{\rho g} + \frac{1}{2} \frac{v_1^2}{g} + Z_1 &= \frac{P_2}{\rho g} + \frac{1}{2} \frac{v_2^2}{g} + Z_2
\end{align*}
\]

Now consider the cyclone

\[
P_1 - P_2 = \Delta P = \text{pressure drop}
\]

\[
\frac{1}{2g} v_1^2 = \frac{1}{2g} v_2^2 = \text{inlet velocity head}
\]

Equivalence of units

\[
\frac{\Delta P}{\rho g} = H_v \left( \frac{1}{2g} v_1^2 \right)
\]

\[
\Delta P = \frac{1}{2} \rho v_i^2 H_v
\]
where \( H_v \) = number of inlet velocity heads equivalent to pressure drop = \( K \frac{HW}{D_e^2} \) \( (D_e = \) exit diameter) \n
\[ K = 16 \text{ (range 12-18) for direct tangential entry} \]
\[ = 7.5 \text{ for inlet vanes} \]

7.5 POWER REQUIRED

\[ \text{BHP} = \dot{\omega}_f \frac{1}{\eta_f \eta_m} \text{ (C&A Eqn.4.14)} \]

where \( \text{BHP} \) = break “horse” power \( (W) \)

\[ \dot{\omega}_f = \text{work of fluid} = Q\Delta P \left( \frac{m^3}{s} \frac{N}{m^2} = \frac{N \cdot m}{s} = W \right) \]

\( \eta_f = \text{efficiency of fan} = \frac{\text{work of fluid}}{\text{shaft work}} \)

\[ \eta_m = \text{efficiency of motor} = \frac{\text{shaft work}}{\text{electrical power to motor}} \]

typical values: 0.6-0.9
7.6 DESIGN PROCEDURE

Example Problem

Given:

- Exhaust gas flow rate = 22,600 m$^3$/h
- Exhaust dust concentration = 4.6 g/m$^3$
- Exhaust gas temperature = 121 °C
- Dust density = 1.5 g/cm$^3$
- Desired emission rate = 600 mg/m$^3$

Particle size distribution as follows:

<table>
<thead>
<tr>
<th>$d_{pi}$ (µm)</th>
<th>Mass Fraction, $m_i$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt;75</td>
<td>6</td>
</tr>
<tr>
<td>67.5</td>
<td>2</td>
</tr>
<tr>
<td>50</td>
<td>6</td>
</tr>
<tr>
<td>35</td>
<td>7</td>
</tr>
<tr>
<td>25</td>
<td>12</td>
</tr>
<tr>
<td>17.5</td>
<td>9</td>
</tr>
<tr>
<td>12.5</td>
<td>14</td>
</tr>
<tr>
<td>8.75</td>
<td>10</td>
</tr>
<tr>
<td>6.25</td>
<td>12</td>
</tr>
<tr>
<td>3.75</td>
<td>14</td>
</tr>
<tr>
<td>1.25</td>
<td>8</td>
</tr>
</tbody>
</table>

Design a cyclone to meet the desired emission rate
1) Calculate desired removal efficiency
\[ \eta_o = \left( \frac{4.6 \text{ g/m}^3 \times 10^3 \text{ mg/g}}{4.6 \times 10^3 \text{ mg/m}^3} \right) - (600 \text{ mg/m}^3) \times 100 = 86.96\% \]

2) Assume an inlet velocity
\[ V_i = 50 \text{ ft/s} = 15.2 \text{ m/s} \]

3) Choose a design (from C&A, Table 4.1)
High efficiency Stairmand design (1) has good removal/\( \Delta P \) ratio according to Koch & Licht (1977).

4) Calculate diameter
\[ Q = H \cdot W \cdot v_i \]
\[ H = 0.5D \]
\[ W = 0.2D \]
\[ Q = (0.5D)(0.2D) \times v_i = 0.1D^2 v_i \]
\[ D = \sqrt{\frac{Q}{0.1v_i}} = \sqrt{\frac{(22600 \text{ m}^3/\text{h})}{0.1(15.2 \text{ m/s})(3600 \text{s/h})}} = 2.03 \text{m} \]

5) Calculate \( d_{pc} \)
\[ d_{pc} = \sqrt{\frac{9\mu W}{2\pi Ne v_i \rho_p}} \]
T = 120°C
\[ \mu = 0.055 \text{ lbm/h⋅ft} = 2.27\times10^{-5} \text{ kg/m⋅s} \]
\[ \rho = 0.0558 \text{ lbm/ft}^3 = 0.894 \text{ kg/m}^3 \]
(C&A Table B.2)
\[ W = 0.2D = 0.2(2.03 \text{m}) = 0.406 \text{ m} \]
\[
\text{Ne} = \frac{1}{H} \left[ L_b + \frac{L_c}{2} \right] = \frac{D}{H} \left[ \frac{1}{D} \left( L_b + \frac{L_c}{2} \right) \right] = \frac{D}{H} \left[ \frac{L_b}{D} + \frac{L_c}{2D^2} \right] = \frac{1}{0.5} \left[ 1.5 + \frac{2.5}{2} \right] = 5.5 \text{ turns}
\]

\[ \rho_p = 1500 \text{ kg/m}^3 \]

\[ d_{pc} = \sqrt{\frac{9(2.27 \times 10^{-5} \text{ kg / m} \cdot \text{s})(0.406 \text{ m})}{2\pi(5.5)(15.2 \text{ m} / \text{s})(1500 \text{ kg / m}^3)}} = 10.3 \mu\text{m} \]

6) Calculate \( \eta_o \)

\[ \eta_i = \frac{1}{1 + \left( \frac{d_{pc}}{d_{pi}} \right)^2} = \frac{1}{1 + \left( \frac{10.3 \mu\text{m}}{d_{pi}} \right)^2} \]

<table>
<thead>
<tr>
<th>( d_{pi} ) (( \mu\text{m} ))</th>
<th>Mass Fraction, ( m_i ) (%)</th>
<th>Grade Removal. Efficiency, ( \eta_i )</th>
<th>Grade Rem., ( m_i \eta_i )</th>
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</thead>
<tbody>
<tr>
<td>&gt;75</td>
<td>6</td>
<td>100</td>
<td>600</td>
</tr>
<tr>
<td>67.5</td>
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<td>97.7</td>
<td>195</td>
</tr>
<tr>
<td>50</td>
<td>6</td>
<td>95.9</td>
<td>576</td>
</tr>
<tr>
<td>35</td>
<td>7</td>
<td>92.0</td>
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</tr>
<tr>
<td>8.75</td>
<td>10</td>
<td>41.9</td>
<td>419</td>
</tr>
<tr>
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<td>26.9</td>
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<td>164</td>
</tr>
<tr>
<td>1.25</td>
<td>8</td>
<td>1.5</td>
<td>12</td>
</tr>
<tr>
<td>( \Sigma )</td>
<td></td>
<td>5456</td>
<td></td>
</tr>
</tbody>
</table>
7) Calculate $\Delta P$

$$H_v = K \frac{HW}{De^2} = K \frac{(H/D)(W/D)}{(De/D)^2} = 16 \frac{(0.5)(0.2)}{(0.5)^2} = 6.4$$

$$\Delta P = \frac{1}{2} \rho v_i^2 H_v = \frac{1}{2} (0.894 \text{kg/m}^3) 6.4 (15.2 \text{m/s})^2$$

$$= 66.1 \text{kg/m}^2 \frac{1}{s^2} \frac{1}{m^2} = 661 \text{Pa} = 2.6 \text{ in water}$$

(Allowable 250-4000Pa, 1-16 inches H$_2$O)

8) Adjust $v_i$ or no. of cyclones

$\eta$ too high $\rightarrow$ decrease $v_i$

decrease no. of cyclones

increase $D$

$\rightarrow$ recalculate $\eta$, $\Delta P$

$\eta$ too low $\rightarrow$ increase $v_i$

increase no. of cyclones

decrease $D$

$\rightarrow$ recalculate $\eta$, $\Delta P$

$\eta_o = 54.6 < 87$, need $d_{pc} \cong 2.2 \mu m$ for $\eta_o = 87$

(See spreadsheet on page 10)

Use formula

$$d_{pc} = \sqrt{\frac{9\mu W}{2\pi N v_i \rho}} = 2.2 \times 10^{-6} \text{ m} \quad \text{to find}$$
W = \frac{2\pi d_{pc}^2 Nev_i \rho_p}{9 \mu} = \frac{2\pi \times (2.2e^{-6})^2 \times 5.5(15.2) \times 1500}{9 \times (2.27 \times 10^{-5})} = 19\text{mm}

W/D = 0.2, D = W/0.2 = 93\text{mm}

In order to maintain same \(v_i\), need to increase no. of cyclones:

\[
Q = \frac{\frac{A}{Q_i}}{\frac{\frac{\Delta P}{H}}{W}}
\]

\[
N = \frac{Q}{HWv_i} = \frac{(22600 \text{m}^3/\text{h})(1/3600 \text{h}/\text{s})}{0.5(93 \times 10^{-3})0.2(93 \times 10^{-3} \text{m})15.2 \text{m/s}} = 475
\]

too high, try \(\uparrow v_i\) to bring \(\Delta P\) to max

\[
\Delta P_{\text{max}} = 20'' \text{H}_2\text{O} = 4980 \text{ Pa}
\]

\[
v_{i,\text{max}} = \sqrt{\frac{2\Delta P}{\rho H_v}} = 41.7 \text{m/s}
\]

\[
W = \frac{2\pi d_{pc}^2 Nev_i \rho_p}{9 \mu} = \frac{2\pi \times (2.2e^{-6})^2 \times 5.5(41.7) \times 1500}{9 \times (2.27 \times 10^{-5})} = 51\text{mm}
\]

D = W/0.2 = 255 mm

\[
N = \frac{Q}{HWv_i} = \frac{(22600 \text{m}^3/\text{h})(1/3600 \text{h}/\text{s})}{0.5(255 \times 10^{-3})0.2(255 \times 10^{-3} \text{m})41.7 \text{m/s}} = 23
\]

\[
\therefore \text{use 23 cyclones}
\]
9) Check Saltation Velocity

\[ \omega = \left[ \frac{4g\mu (\rho_p - \rho_f)}{3\rho_f^2} \right]^{\frac{1}{3}} \]

\[ = \left[ \frac{4(32.2 \text{ft/s}^2 \times 1.528 \times 10^{-5} \text{lb/ft}^3 (93.6 \text{lb/ft}^3 - 0.0558 \text{lb/ft}^3))}{3(0.0558 \text{lb/ft}^3)} \right]^{\frac{1}{3}} \]

\[ = 2.70 \text{ft/s} \]

with 474 cyclones: \( v_i = 50 \text{ ft/s}, D=93 \text{ mm}, W=19 \text{ mm} \)

\[ v_s = 2.055 \omega \frac{(W/D)^{0.4}}{(1-W/D)^{0.32}} D^{0.067} v_i^{0.667} \]

\[ = 2.055(2.7 \text{ft/s}) \frac{0.2^{0.4}}{(1-0.2)^{0.32}} (0.305 \text{ft})^{0.067} (50 \text{ft})^{0.667} \]

\[ = 39.3 \text{ft/s} \]

\[ \frac{v_i}{v_s} = \frac{50 \text{ ft/s}}{39.3 \text{ ft/s}} = 1.27, \quad \text{good, since max } \eta @ 1.25 \]

with 23 cyclones: \( v_i = 41.7 \text{ m/s} = 137 \text{ ft/s}, D=255 \text{ mm}, W=51 \text{ mm} \)

\[ \frac{v_i}{v_s} = \frac{137 \text{ ft/s}}{82.36 \text{ ft/s}} \]

\[ = 1.66 \]

a bit too high

a closer look at \( v_s \) equation: \( \omega \) same, \( v_i^{0.667} \uparrow \), or \( D^{0.067} \uparrow \), \( v_s \uparrow \)

link \( D \) with \( v_i \), \( \frac{v_i}{v_s} = 0.45v_i^{0.266} \)

if increase \( D \) to keep \( v_i \) at 50 ft/s, \( \eta \) will decrease

need to use spreadsheet for optimal design
Design criteria:

- Removal efficiency
- Pressure drop

Variable parameters:

- Inlet velocity
- Number of cyclones
- Size (H, W, D, but only one independent)

Relative effect of centrifugal force

\[ F_c = \frac{m_p v_i^2}{g_c R} \]
\[ F_g = \frac{m_p g}{g_c} \]
\[ \frac{F_c}{F_g} = \frac{m_p v_i / g_c R}{m_p g / g_c} = \frac{v_i^2}{g_c R} \]
\[ = \frac{41.7^2}{9.81(0.093/2)} = 3812 \]

10) Energy use

\[ BHP = Q \Delta P \frac{1}{\eta_f} \frac{1}{\eta_m} = \frac{22600 \text{m}^3 / \text{h}}{3600 \text{s} / \text{h}} \frac{661 \text{N} / \text{m}^2}{0.6 \times 0.9} \]
\[ = 7684 \text{W} = 7.7 \text{kW} \]

Over 1 day, the energy cost is 24h (7.7kW) ($0.07/kWh) = $13